

20 December 1994

HEP-PH/9501360
INTERNAL REPORT
PAR/LPTHE/94-44

SU(2) HEAVY FLAVOUR SYMMETRY
FOR $B \rightarrow K$ (K^*) HADRONIC FORM FACTORS

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Abstract

We show that the factorization assumption in colour-suppressed B meson decays is not ruled out by experimental data on $B \rightarrow K(K^*) + J/\Psi(\Psi')$. The problem previously pointed out might be due to an inadequate choice of hadronic form factors.

Within the Isgur-Wise $SU(2)$ heavy flavour symmetry framework, we search for possible q^2 -dependences of form factors that satisfy both the large longitudinal polarization ρ_L observed in $B \rightarrow K^* + J/\Psi$ and the relatively small ratio of rates $R_{J/\Psi} = \Gamma(B \rightarrow K^* + J/\Psi)/\Gamma(B \rightarrow K + J/\Psi)$.

We find out that the puzzle could be essentially understood if the $A_1(q^2)$ form factor is frankly decreasing (instead of being almost constant or increasing as commonly assumed).

Of course, the possibility of understanding experimental data is not necessarily a proof of factorization.

PACS index 13. 25. Hw, 14. 40. Nd

Contents

I	Introduction	3
II	The Isgur-Wise relations due to SU(2) heavy flavour symmetry	6
III	Factorization and Kinematics, Experimental Data	14
IV	The decay modes $B \rightarrow K(K^*) + \Psi'$	18
V	The decay modes $B \rightarrow K(K^*) + J/\Psi$	21
VI	The decay modes $B \rightarrow K(K^*) + \eta_c$	26
VII	Comparison of Different Charmonium States Production	28
VIII	Radiative Decay $B \rightarrow K^* + \gamma$	34
IX	$D \rightarrow \bar{K}(\bar{K}^*)$ Hadronic Form Factors	36
X	Critical Discussions and Conclusions	43

I Introduction

In a recent letter [1], Kamal and two of us (M.G. and X.Y.P.) have shown, within the factorization approach, the failure of commonly used $B \rightarrow K(K^*)$ form factors in explaining recent data on $B \rightarrow J/\Psi + K(K^*)$ decays. The main problem is a simultaneous fit of the large fractional longitudinal polarization ρ_L in $B \rightarrow J/\Psi + K^*$ decay and of the relatively small ratio of rates $R_{J/\Psi}$ between $J/\Psi + K^*$ and $J/\Psi + K$ in the final states. We concluded that this difficulty in understanding experimental data might be due to a failure of the factorization method or to a wrong choice of hadronic form factors or both.

Such an analysis has been independently performed by Aleksan, Le Yaouanc, Oliver, Pène and Raynal [2] who also found difficulties in explaining both ρ_L and $R_{J/\Psi}$, in spite of their large choice of heavy to light hadronic form factors consistent with asymptotic scaling law [3].

In our previous work [1], in addition to our exploration of the usual $B \rightarrow K(K^*)$ form factors available in the literature, we also related the $B \rightarrow K(K^*)$ to the $D \rightarrow K(K^*)$ form factors using the SU(2) flavour symmetry between the b and c quarks as first proposed by Isgur and Wise [3]. The input data are the hadronic form factors in the D sector at $q^2 = 0$ as extracted from the semi-leptonic $D \rightarrow \bar{K}(\bar{K}^*) + \ell^+ + \nu_\ell$ decays. In such experiments, the q^2 distribution has not been measured and the analysis of experimental data has been made assuming monopole q^2 -dependence for all the $D \rightarrow \bar{K}(\bar{K}^*)$ form factors. For that reason in [1], we have also used monopole forms in the B sector. The resulting $B \rightarrow K(K^*)$ form factors obtained in this way are also unable to explain simultaneously experimental data on ρ_L and $R_{J/\Psi}$.

Our method, based on the Isgur-Wise relations, has been subsequently adopted by Cheng and Tseng [4] who considered various types of q^2 dependences for the hadronic form factors. However their model still encounters difficulties in reproducing correctly experimental data.

The purpose of this paper is to make a purely phenomenological investigation of the possible q^2 -dependence – we shall call scenario – of the hadronic form factors in the B sector such that, assuming factorization and using the Isgur-Wise relations [3] together with the latest data at $q^2 = 0$ in the D sector [5], we can obtain a good fit for both ρ_L and $R_{J/\Psi}$.

Some preliminary remarks are in order. We are aware of the fact that the values at $q^2 = 0$

of the $D \rightarrow K(K^*)$ form factors have been extracted from semi-leptonic decay experiments assuming a monopole q^2 -dependence for all hadronic form factors. This ansatz is certainly inconsistent with all theoretical expectations coming, for instance, from QCD sum rules [6], from lattice gauge calculations [7] as well as from scaling law of heavy flavours [2, 3, 4]. A correct procedure would be to reanalyze the triple angular distribution fit in D semi-leptonic decay, with different scenarios, in order to evaluate the sensitivity to the scenarios of the normalization at $q^2 = 0$ of the form factors. Such a study has not yet been done by experimentalists. Due to the limited range of q^2 in the D semi-leptonic decays, it was implicitly assumed that the values at $q^2 = 0$ of the form factors could well be insensitive to the scenarios. To clarify and settle the issue, the cleanest information would come from measurements of the q^2 distributions for the rates and for the various polarizations in the semi-leptonic D sector. We are still far from such an ideal situation and for the time being, the only pragmatic way is to use the results quoted in [5] with errors included for the values at $q^2 = 0$ of the form factors.

We propose, in this paper, four types of scenarios for each of the $B \rightarrow K(K^*)$ hadronic form factors F_1, A_1, A_2 and V in the Bauer, Stech and Wirbel (BSW henceforth) notation [8].

The q^2 dependences, taken as $(1 - q^2/\Lambda^2)^{-n}$, are applied indiscriminately to all of these form factors. The algebraic integer n symbolically represents n_F, n_1, n_2 and n_V associated respectively to F_1, A_1, A_2 and V . These integers n can take four values corresponding to four types of scenarios mentioned above : -1 for a linear dependence, 0 for a constant, $+1$ for a monopole and $+2$ for a dipole. The respective pole masses $\Lambda_F, \Lambda_1, \Lambda_2$ and Λ_V for F_1, A_1, A_2 and V are treated as phenomenological parameters. Being related, in some way, to bound states of the $\bar{b}s$ system, we impose to these parameters the physical constraint to be in the range $(5 - 6) \text{ GeV}$. Such a requirement is satisfied by the pole masses of the BSW model [8].

We now summarize the results of our finding :

The experimental data on ρ_L and $R_{J/\Psi}$ indeed can be fitted for three scenarios corresponding to three possibilities $n_2 = 2, 1, 0$ for $A_2^{BK^*}$ together with :

- i) $n_1 = -1$ for a linear decreasing with q^2 of $A_1^{BK^*}$
- ii) $n_V = +2$ for a dipole increasing with q^2 of V^{BK^*}
- iii) $n_F = +1$ for a monopole increasing with q^2 of F_1^{BK}

For a given selected scenario we have a non empty allowed domain in the $\Lambda_F, \Lambda_1, \Lambda_2, \Lambda_V$ param-

eter space. Therefore we obtain hadronic form factors for $B \rightarrow K(K^*)$ reproducing correctly (within experimental errors) ρ_L and $R_{J/\Psi}$ with the parameters $\Lambda_F, \Lambda_1, \Lambda_2, \Lambda_V$ physically acceptable. We now easily understand why previous attempts [1, 2, 4] were unsuccessful, mainly because the decrease with q^2 of the form factor $A_1(q^2)$ has never been seriously considered. Let us emphasize however that such an unusual q^2 behaviour has already been obtained by Narison [6] in the QCD sum rule approach. Of course our result is not a proof of factorization in the B sector. It only makes wrong the statement that the difficulty in fitting simultaneously ρ_L and $R_{J/\Psi}$ implies that factorization breaks down in colour-suppressed B decays.

This paper is organized as follows. In part II we discuss in some detail the Isgur-Wise relations [3] and, in particular, the consistency of scenarios in the B and D sectors as well as the relations between the parameters Λ_j ($j = 1, 2, V, F$) in both sectors. The case of the form factors F_0 and A_0 , associated to the spin zero part of the currents is equally discussed.

In part III we give the consequences of factorization for the decay amplitudes $B \rightarrow K(K^*) + (\eta_c, J/\Psi, \Psi')$ which are colour-suppressed processes. We study the kinematics and we review the available experimental data for these decay modes.

The part IV is devoted to the decay modes $B \rightarrow K(K^*) + \Psi'$, in which some scenario-independent results can be obtained. The left-right asymmetry \mathcal{A}'_{LR} between the two transverse polarizations in $B \rightarrow K^* + \Psi'$ is found to be large and close to its maximal value. The fractional longitudinal polarization ρ'_L turns out to be a slowly varying function of Λ_2 and the ratio of rates $R_{\Psi'}$ a function of Λ_2 and Λ_F . The result for $R_{\Psi'}$ is consistent with experiment [5]. Our prediction for ρ'_L is compared with that of Kamal and Sanda [9] who use seven different scenarios.

The part V is the central part of this paper being related to the decay modes $B \rightarrow K(K^*) + J/\Psi$. The study of ρ_L and $R_{J/\Psi}$ allows us to select only three surviving scenarios among the $4^3 = 64$ possible cases and to constraint the $\Lambda_F, \Lambda_1, \Lambda_2, \Lambda_V$ parameter space.

In part VI we make predictions for the decay modes $B \rightarrow K(K^*) + \eta_c$ for which, unfortunately no experimental data are available.

The comparison of various charmonium states involves different leptonic decay constants $f_{\Psi'}, f_{J/\Psi}$ and f_{η_c} . This problem is studied in part VII in relation with previous works [9] and [10] and with experimental data available only for the ratios of Ψ' and J/Ψ production.

The $B \rightarrow K^*$ vector and axial vector form factors studied here can be related under reasonable assumptions to the tensor and pseudotensor $B \rightarrow K^*$ form factors describing the radiative decay $B \rightarrow K^* + \gamma$ recently observed. This problem is briefly discussed in part VIII.

Finally in part IX, we come back to the D sector in the light of results obtained in the B sector. Of course, the hadronic form factors F_1, A_1, A_2, V follow the same scenarios in the B and D sectors with pole masses related by Equations (II.24) and (II.37). We determine the normalized q^2 distributions for semi-leptonic decays $D \rightarrow \bar{K} \ell^+ \nu_\ell$, $D \rightarrow \bar{K}^* \ell^+ \nu_\ell$ and for this last mode the integrated longitudinal polarization ρ_L^{sl} and the left-right asymmetry \mathcal{A}_{LR}^{sl} between transverse polarizations (sl stands for semi-leptonic). We also study the hadronic two body decay modes $D^0 \rightarrow K^-(K^{*-}) + \pi^+(\rho^+)$ which are colour favoured and we show that simple factorization assumption fails in the D sector where, in addition, final state strong interactions play an important role.

Discussions and critical remarks on the results are given in the conclusion.

II The Isgur-Wise relations due to SU(2) heavy flavour symmetry

1°) The SU(2) flavour symmetry between the heavy b and c quarks allows us to derive relations between the $B \rightarrow K(K^*)$ and $D \rightarrow K(K^*)$ hadronic form factors at the same velocity transfer though at different momentum transfers. Calling t_B (t_D) the value of the squared momentum transfer q^2 for $B(D)$ form factors, we obtain the following kinematic relations :

$$v_b \cdot v_K = v_c \cdot v_K \quad \text{or} \quad m_c t_B - m_b t_D - (m_b - m_c)(m_b - m_c - m_K^2) = 0 \quad (\text{II.1})$$

$$v_b \cdot v_{K^*} = v_c \cdot v_{K^*} \quad \text{or} \quad m_c t_B^* - m_b t_D^* - (m_b - m_c)(m_b - m_c - m_{K^*}^2) = 0 \quad (\text{II.2})$$

In particular, at zero squared momentum transfer in the D sector, $t_D = t_D^* = 0$, we get

$$t_B \equiv t_B^o = \left(\frac{m_b}{m_c} - 1 \right) (m_b - m_c - m_K^2) \quad (\text{II.3})$$

$$t_B^* \equiv t_B^{*o} = \left(\frac{m_b}{m_c} - 1 \right) (m_b - m_c - m_{K^*}^2) \quad (\text{II.4})$$

and useful relations such as

$$t_D = \frac{m_c}{m_b} (t_B - t_B^o) \quad (\text{II.5})$$

$$t_D^* = \frac{m_c}{m_b} (t_B^* - t_B^{o*}) \quad (\text{II.6})$$

The knowledge of the hadronic form factors at $q^2 = 0$ in the D sector will determine the hadronic form factors in the B sector at $q^2 = t_B^o$ for the $B \rightarrow K$ case and at $q^2 = t_B^{*o}$ for the $B \rightarrow K^*$ case.

The values of t_B^o and t_B^{*o} depend on the quark masses m_b and m_c . We choose, in this paper, $m_b = 4.7 \text{ GeV}$, $m_c = 1.45 \text{ GeV}$ and get :

$$t_B^o = 14.7287 \text{ GeV}^2 \quad (\text{II.7})$$

$$t_B^{*o} = 13.4933 \text{ GeV}^2 \quad (\text{II.8})$$

2^o) We first consider the case of $B \rightarrow K$ and $D \rightarrow K$ form factors. The matrix elements of the weak current involve two form factors f_+ and f_- defined by

$$\langle K | J_\mu | D \rangle = (p_D + p_K)_\mu f_+^{DK}(q^2) + (p_D - p_K)_\mu f_-^{DK}(q^2) \quad (\text{II.9})$$

$$\langle K | J_\mu | B \rangle = (p_B + p_K)_\mu f_+^{BK}(q^2) + (p_B - p_K)_\mu f_-^{BK}(q^2) \quad (\text{II.10})$$

where $q = p_{B,D} - p_K$.

The Isgur-Wise relations are written as [3]

$$(f_+ + f_-)^{BK}(t_B) = C_{bc} \sqrt{\frac{m_c}{m_b}} (f_+ + f_-)^{DK}(t_D) \quad (\text{II.11})$$

$$(f_+ - f_-)^{BK}(t_B) = C_{bc} \sqrt{\frac{m_b}{m_c}} (f_+ - f_-)^{DK}(t_D) \quad (\text{II.12})$$

or equivalently

$$f_\pm^{BK}(t_B) = C_{bc} \left[\frac{m_b + m_c}{2\sqrt{m_b m_c}} f_\pm^{DK}(t_D) - \frac{m_b - m_c}{2\sqrt{m_b m_c}} f_\mp^{DK}(t_D) \right] \quad (\text{II.13})$$

where $C_{bc} \equiv [\frac{\alpha_s(m_b)}{\alpha_s(m_c)}]^{-6/25}$ is the QCD correction factor. [‡]

In the BSW basis [8], the spin one and the spin zero parts of the weak current are separated and two new form factors F_1 and F_0 are defined

$$F_1^{PK}(q^2) = f_+^{PK}(q^2) \quad (\text{II.14})$$

$$F_0^{PK}(q^2) = f_+^{PK}(q^2) + \frac{q^2}{m_P^2 - m_K^2} f_-^{PK}(q^2) \quad (\text{II.15})$$

[‡]In this paper, we use $C_{bc} \equiv [\frac{\alpha_s(m_b)}{\alpha_s(m_c)}]^{-6/25} \simeq 1.135$ which has been obtained by using the recent world averaged values for $\Lambda_{\overline{MS}} : \Lambda_{\overline{MS}}^5 = (225 \pm 85) \text{ MeV}$, $\Lambda_{\overline{MS}}^4 = (325 \pm 110) \text{ MeV}$, and for the quark masses : $m_b = 4.7 \text{ GeV}$, $m_c = 1.45 \text{ GeV}$.

where $P = B$ or D .

Let us define the ratio of form factors f_- and f_+ by :

$$\mu^P(q^2) = - \frac{f_-^{PK}(q^2)}{f_+^{PK}(q^2)} \quad P = B, D \quad (\text{II.16})$$

Using Eq.(II.13), we obtain a relation between the two quantities $\mu^B(t_B)$ and $\mu^D(t_D)$

$$\mu^B(t_B) = \frac{\sigma + \mu^D(t_D)}{1 + \sigma \mu^D(t_D)} \quad (\text{II.17})$$

where the constant σ depends only on the quark masses

$$\sigma = \frac{m_b - m_c}{m_b + m_c} \quad (\text{II.18})$$

The spin one function F_1^{BK} is now related to f_+^{DK} and μ^D , and we obtain from Eq.(II.13)

$$F_1^{BK}(t_B) = C_{bc} \frac{m_b + m_c}{2\sqrt{m_b m_c}} [1 + \sigma \mu^D(t_D)] F_1^{DK}(t_D) \quad (\text{II.19})$$

and for the spin zero function F_0^{BK} , we get from Eq.(II.15)

$$F_0^{BK}(q^2) = [1 - \frac{q^2}{m_B^2 - m_K^2} \mu^B(q^2)] F_1^{BK}(q^2) \quad (\text{II.20})$$

3^o) As explained previously, we shall use in the D sector the values of the hadronic form factors at $q^2 = 0$ as coming from semi-leptonic experimental data. Because of the normalization constraint $F_1^{DK}(0) = F_0^{DK}(0)$ only $f_+^{DK}(0)$ is known and we cannot have, in that way, any information on $f_-^{DK}(0)$ and $\mu^D(0)$ § .

We now make an assumption which is natural in the framework of the SU(2) heavy flavour symmetry. If $f_+^{DK}(q^2)$ and $f_-^{DK}(q^2)$ have the same type of q^2 -dependence, no matter how it is, then the ratio μ^D is constant and, using the Isgur-Wise relations (II.11) and (II.12) we easily see that the same property extends to the B sector and , in particular, the ratio μ^B is a constant related to μ^D by Eq.(II.17). Of course, $F_1^{BK}(q^2)$ and $F_1^{DK}(q^2)$ have the same type of q^2 dependence.

For instance, if $F_1^{DK}(q^2)$ is written in the form

$$F_1^{DK}(q^2) = \frac{F_1^{DK}(0)}{[1 - \frac{q^2}{\Lambda_{DF}^2}]^{n_F}} \quad (\text{II.21})$$

§ In principle, $f_-^{DK}(0)$ and $\mu^D(0)$ could be measured in $D \rightarrow \bar{K} + \mu^+ + \nu_\mu$ namely by looking at polarized muon.

where n_F is some algebraic integer, then, using Eq.(II.19) we obtain a similar expression for $F_1^{BK}(q^2)$

$$F_1^{BK}(q^2) = \frac{F_1^{BK}(0)}{[1 - \frac{q^2}{\Lambda_F^2}]^{n_F}} \quad (\text{II.22})$$

with the same n_F .

Furthermore a relation between the pole masses Λ_F and Λ_{DF} has been previously given in Ref.[1] and it comes from the identity

$$(1 - \frac{t_D}{\Lambda_{DF}^2}) = 1 - \frac{m_c}{m_b} \frac{t_B - t_B^o}{\Lambda_{DF}^2} = (1 + \frac{m_c}{m_b} \frac{t_B^o}{\Lambda_{DF}^2}) (1 - \frac{t_B}{\Lambda_F^2}) \quad (\text{II.23})$$

The result is

$$m_c \Lambda_F^2 - m_b \Lambda_{DF}^2 = m_c t_B^o = (m_b - m_c)(m_b - m_c - m_K^2) \quad (\text{II.24})$$

The values at $q^2 = 0$ of the form factor F_1 in the B and D sectors are also related by Eq.(II.19).

The result depends on n_F and is given by

$$\frac{F_1^{BK}(0)}{F_1^{DK}(0)} = C_{bc} \left(\frac{m_b + m_c}{2\sqrt{m_b m_c}} [1 + \sigma \mu^D] \left[\frac{m_b \Lambda_{DF}^2}{m_c \Lambda_F^2} \right]^{n_F} \right) \quad (\text{II.25})$$

4^o) An interesting scenario for F_1^{DK} , suggested by many theoretical studies [2, 8, 11] as well as supported by experimental data [5, 12] is a monopole dependence $n_F = 1$ with a pole mass Λ_{DF} in the 2 GeV region. While F_1^{BK} and F_1^{DK} increase with q^2 by a monopole type, from Eq.(II.20) we easily see that the form factors F_0^{BK} and F_0^{DK} have a different q^2 behaviour due to a supplementary linearly decreasing factor in Eq.(II.20)

$$\begin{aligned} & \left[1 - \frac{q^2}{m_B^2 - m_K^2} \mu^B \right] \quad \text{for} \quad F_0^{BK} \\ & \left[1 - \frac{q^2}{m_D^2 - m_K^2} \mu^D \right] \quad \text{for} \quad F_0^{DK} \end{aligned}$$

In fact, for a particular value of μ^B or μ^D , this factor can exactly cancel the pole of F_1 in the B or in the D sector, thus making F_0 constant. This is the case when

$$\mu^B = \frac{m_B^2 - m_K^2}{\Lambda_F^2} \quad \text{or} \quad \mu^D = \frac{m_D^2 - m_K^2}{\Lambda_{DF}^2} \quad (\text{II.26})$$

In this way, the value of $\mu^B(\mu^D)$ is related to the pole mass $\Lambda_F(\Lambda_{DF})$. However such a situation, in general, does not occur in both B and D sectors because Λ_F^2 and Λ_{DF}^2 are not independent from each other due to Eq.(II.24).

If we impose the constraints (II.26) for μ^B and μ^D , we can determine the pole masses Λ_F^2 and Λ_{DF}^2 . The relations (II.17), (II.24) and (II.26) lead to a second order equation for Λ_{DF}^2 , the roots of which are real. However one root (negative) is physically unacceptable, and the other one (positive) gives Λ_{DF} within the $(2 - 3) \text{ GeV}$ range, corresponding to Λ_F within the $(5 - 6) \text{ GeV}$ range and the value of μ^B is in the neighbourhood of 1.

The calculation involves the b and c quark masses : with $m_b = 4.7 \text{ GeV}$ and $m_c = 1.45 \text{ GeV}$, the hadronic form factor F_0 is constant in both B and D sectors for the following pole mass values $\Lambda_{DF} = 2.32 \text{ GeV}$, $\Lambda_F = 5.67 \text{ GeV}$ and for μ^B and μ^D , we obtain $\mu^B = 0.8585$ and $\mu^D = 0.6041$.

5°) We do not have, to our knowledge, a theoretical estimate of the form factors $f_-^{DK}(0)$ and $f_-^{BK}(0)$. The heavy quark symmetry limit can not be applied for heavy quark to light quark, $b \rightarrow s$, transitions. Therefore we shall use, in the B sector, a model suggested by theoretical considerations [4, 13] where F_0^{BK} is independent of q^2 and F_1^{BK} has a monopole q^2 dependence with a pole mass Λ_F . The parameter μ^B is then a function of Λ_F^2 as given in Eq.(II.26) and the parameter μ^D is known from μ^B by inverting Eq.(II.17)

$$\mu^D = \frac{\mu^B - \sigma}{1 - \sigma \mu^B} \quad (\text{II.27})$$

where $\sigma = 0.5285$ for our choice of quark masses.

We plot in Figure 1 the ratios μ^B and μ^D as functions of Λ_F^2 for values of Λ_F in the $(4 - 7) \text{ GeV}$ range. They are parts of hyperbolae with asymptotia parallel to the Λ_F^2 and μ axes. We plot, by a straight line, Eq.(II.24) which relates the pole masses in the B and D sectors. On the same figure, we also show, with dotted points, the quantity $\bar{\mu}^D$ which corresponds to a constant F_0 form factor in the D sector.

6°) We now study the case of the $B \rightarrow K^*$ and $D \rightarrow K^*$ form factors. The matrix elements of the weak current involve four form factors, f, g, a_+ and a_- in the Isgur-Wise basis, or A_1, V, A_2 and A_0 in the BSW basis.

The Isgur-Wise relations for f and g are very simple [3]

$$f^{BK^*}(t_B^*) = C_{bc} \sqrt{\frac{m_b}{m_c}} f^{DK^*}(t_D^*) \quad (\text{II.28})$$

$$g^{BK^*}(t_B^*) = C_{bc} \sqrt{\frac{m_c}{m_b}} g^{DK^*}(t_D^*) \quad (\text{II.29})$$

Using the relations between the Isgur-Wise and BSW bases :

$$f^{PK^*} = (m_P + m_{K^*}) A_1^{PK^*} \quad , \quad g^{PK^*} = \frac{1}{(m_P + m_{K^*})} V^{PK^*} \quad ; \quad P = B, D \quad (\text{II.30})$$

we get

$$A_1^{BK^*}(t_B^*) = C_{bc} \sqrt{\frac{m_b}{m_c}} \frac{m_D + m_{K^*}}{m_B + m_{K^*}} A_1^{DK^*}(t_D^*) \quad (\text{II.31})$$

$$V^{BK^*}(t_B^*) = C_{bc} \sqrt{\frac{m_c}{m_b}} \frac{m_B + m_{K^*}}{m_D + m_{K^*}} V^{DK^*}(t_D^*) \quad (\text{II.32})$$

In particular, defining the ratio

$$y(q^2) \equiv \frac{V(q^2)}{A_1(q^2)} \quad (\text{II.33})$$

the values of y in the B and D sectors, y^B and y^D are related by :

$$y^B(t_B^*) = \frac{m_c}{m_b} \left(\frac{m_B + m_{K^*}}{m_D + m_{K^*}} \right)^2 y^D(t_D^*) \quad (\text{II.34})$$

The dependence with respect to q^2 of A_1 and V in the B and D sectors is preserved by the Isgur-Wise relations (II.31) and (II.32). We choose the form factors $A_1^{BK^*}(q^2)$ and $V^{BK^*}(q^2)$ such as :

$$A_1^{BK^*}(q^2) = \frac{A_1^{BK^*}(0)}{[1 - \frac{q^2}{\Lambda_1^2}]^{n_1}} \quad (\text{II.35})$$

$$V^{BK^*}(q^2) = \frac{V^{BK^*}(0)}{[1 - \frac{q^2}{\Lambda_V^2}]^{n_V}} \quad (\text{II.36})$$

and analogous expressions in the D sectors with pole masses Λ_{D1} and Λ_{DV} related to Λ_1 and Λ_V by a formula similar to Eq.(II.24)

$$m_c \Lambda_B^2 - m_b \Lambda_D^2 = m_c t_B^{o*} = (m_b - m_c)(m_b m_c - m_{K^*}^2) \quad (\text{II.37})$$

where $\Lambda_B = \Lambda_1$ or Λ_V and $\Lambda_D = \Lambda_{D1}$ or Λ_{DV} . The algebraic integers n_1 and n_V will be restricted to the values $-1, 0, 1$ and 2 . The normalizations at $q^2 = 0$ of these form factors in the B and D sectors are related using Eqs.(II.31) and (II.32)

$$\frac{A_1^{BK^*}(0)}{A_1^{DK^*}(0)} = C_{bc} \sqrt{\frac{m_b}{m_c}} \frac{m_D + m_{K^*}}{m_B + m_{K^*}} \left[1 - \frac{t_B^{*o}}{\Lambda_1^2} \right]^{n_1} \quad (\text{II.38})$$

$$\frac{V^{BK^*}(0)}{V^{DK^*}(0)} = C_{bc} \sqrt{\frac{m_c}{m_b}} \frac{m_B + m_{K^*}}{m_D + m_{K^*}} \left[1 - \frac{t_B^{*o}}{\Lambda_V^2}\right]^{n_V} \quad (\text{II.39})$$

7^o) For the two other form factors a_+ and a_- describing the $P \rightarrow K^*$ transition, the situation is formally similar to f_+ and f_- in the $P \rightarrow K$ case previously considered. The Isgur-Wise relations are written as [3]

$$(a_+ + a_-)^{BK^*}(t_B^*) = C_{bc} \left(\frac{m_c}{m_b}\right)^{3/2} (a_+ + a_-)^{DK^*}(t_D^*) \quad (\text{II.40})$$

$$(a_+ - a_-)^{BK^*}(t_B^*) = C_{bc} \left(\frac{m_c}{m_b}\right)^{1/2} (a_+ - a_-)^{DK^*}(t_D^*) \quad (\text{II.41})$$

from which we deduce

$$a_{\pm}^{BK^*}(t_B^*) = \frac{1}{2} C_{bc} \sqrt{\frac{m_c}{m_b}} \left[\left(1 + \frac{m_c}{m_b}\right) a_{\pm}^{DK^*}(t_D^*) - \left(1 - \frac{m_c}{m_b}\right) a_{\mp}^{DK^*}(t_D^*) \right] \quad (\text{II.42})$$

In the BSW basis [8], the spin one and the spin zero parts of the weak current are separated, with A_1, V and A_2 for the spin one part and A_0 for the spin zero part. The form factors A_2 and A_0 are linear combinations of the form factors a_+ , a_- in the Isgur-Wise basis :

$$A_2^{PK^*}(q^2) = -(m_P + m_{K^*}) a_+^{PK^*}(q^2) \quad (\text{II.43})$$

$$A_0^{PK^*}(q^2) = \frac{m_P + m_{K^*}}{2 m_{K^*}} A_1^{PK^*}(q^2) - \frac{m_P - m_{K^*}}{2 m_{K^*}} A_2^{PK^*}(q^2) + \frac{q^2}{2 m_{K^*}} a_-^{PK^*}(q^2) \quad (\text{II.44})$$

where $P = B$ or D .

At $q^2 = 0$ the normalization of A_0 is determined by $A_1(0)$ and $A_2(0)$:

$$A_0^{PK^*}(0) = \frac{m_P + m_{K^*}}{2 m_{K^*}} A_1^{PK^*}(0) - \frac{m_P - m_{K^*}}{2 m_{K^*}} A_2^{PK^*}(0), \quad P = B, D \quad (\text{II.45})$$

and we can not have any information on the value at $q^2 = 0$ of the form factor a_- . ¶ Let us now define the ratio of the form factors a_- and a_+ by :

$$\lambda^P(q^2) = -\frac{a_-^{PK^*}(q^2)}{a_+^{PK^*}(q^2)} \quad P = B, D \quad (\text{II.46})$$

Using Eq.(II.42) we obtain a relation between the two quantities $\lambda^B(t_B^*)$ and $\lambda^D(t_D^*)$ similar to Eq.(II.17)

$$\lambda^B(t_B^*) = \frac{\sigma + \lambda^D(t_D^*)}{1 + \sigma \lambda^D(t_D^*)} \quad (\text{II.47})$$

¶ similar to the $f_-(q^2)$ case, the form factor $a_-(q^2)$ can be measured in $D \rightarrow \overline{K}^* + \mu^+ + \nu_\mu$.

with σ defined in Eq.(II.18).

The function $A_2^{BK^*}(t_B^*)$ is now expressed in terms of $A_2^{DK^*}(t_D^*)$ and $\lambda^D(t_D^*)$.

Using Eqs.(II.42) and (II.43), we get

$$A_2^{BK^*}(t_B^*) = \frac{1}{2} C_{bc} \sqrt{\frac{m_c}{m_b}} \left(1 + \frac{m_c}{m_b}\right) \frac{m_B + m_{K^*}}{m_D + m_{K^*}} [1 + \sigma \lambda^D(t_D^*)] A_2^{DK^*}(t_D^*) \quad (\text{II.48})$$

With Eqs.(II.44) and (II.46) we obtain the spin zero function $A_0^{BK^*}$:

$$A_0^{BK^*}(q^2) = \frac{m_B + m_{K^*}}{2 m_{K^*}} A_1^{BK^*}(q^2) - \frac{m_B - m_{K^*}}{2 m_{K^*}} \left[1 - \frac{q^2}{m_B^2 - m_{K^*}^2} \lambda^B(q^2)\right] A_2^{BK^*}(q^2) \quad (\text{II.49})$$

Defining the ratio :

$$x(q^2) \equiv \frac{A_2(q^2)}{A_1(q^2)} \quad (\text{II.50})$$

and using Eqs.(II.31) and (II.48), the values of x in the B and D sectors, x^B and x^D , are related by

$$x^B(t_B^*) = \frac{1}{2} \frac{m_c}{m_b} \left(1 + \frac{m_c}{m_b}\right) \left(\frac{m_B + m_{K^*}}{m_D + m_{K^*}}\right)^2 [1 + \sigma \lambda^D(t_D^*)] x^D(t_D^*) \quad (\text{II.51})$$

8^o) We now make an assumption for a_+ and a_- similar to the one made previously for f_+ and f_- . If $a_+^{DK^*}(q^2)$ and $a_-^{DK^*}(q^2)$ have the same type of q^2 -dependence, no matter how it is, then the ratio λ^D is constant and using the Isgur-Wise relations (II.40) and (II.41), the same property extends to the B sector with, in particular, a constant value for the ratio λ^B related to λ^D by Eq.(II.47). Of course, $A_2^{BK^*}(q^2)$ and $A_2^{DK^*}(q^2)$ have also the same type of q^2 -dependence.

The form factor $A_2^{BK^*}(q^2)$ is written in the form

$$A_2^{BK^*}(q^2) = \frac{A_2^{BK^*}(0)}{\left[1 - \frac{q^2}{\Lambda_2^2}\right]^{n_2}} \quad (\text{II.52})$$

and the normalizations at $q^2 = 0$ of A_2 in the B and D sectors are related using Eq.(II.48)

$$\frac{A_2^{BK^*}(0)}{A_2^{DK^*}(0)} = \frac{1}{2} \sqrt{\frac{m_c}{m_b}} C_{bc} \left(1 + \frac{m_c}{m_b}\right) \frac{m_B + m_{K^*}}{m_D + m_{K^*}} [1 + \sigma \lambda^D] \left[1 - \frac{t_B^{*0}}{\Lambda_2^2}\right]^{n_2} \quad (\text{II.53})$$

The second term in Eq.(II.49) for A_0 is written as

$$A_0^{BK^*}(q^2) - \frac{m_B + m_{K^*}}{2 m_{K^*}} A_1^{BK^*}(q^2) = - \frac{m_B - m_{K^*}}{2 m_{K^*}} \left[1 - \frac{q^2}{m_B^2 - m_{K^*}^2} \lambda^B\right] \frac{A_2^{BK^*}(0)}{\left[1 - \frac{q^2}{\Lambda_2^2}\right]^{n_2}} \quad (\text{II.54})$$

and it exhibits a different q^2 behaviour compared to A_2 due to a supplementary linearly decreasing factor $(1 - \frac{q^2 \lambda^B}{m_B^2 - m_{K^*}^2})$. For a particular value of λ^B or λ^D , this factor cancels exactly

one power of $A_2^{BK^*}$ or $A_2^{DK^*}$. This is the case when

$$\lambda^B = \frac{m_B^2 - m_{K^*}^2}{\Lambda_2^2} \quad or \quad \lambda^D = \frac{m_D^2 - m_{K^*}^2}{\Lambda_{D2}^2} \quad (\text{II.55})$$

In this way the value of $\lambda^B(\lambda^D)$ is related to the pole mass $\Lambda_2(\Lambda_{D2})$. However such a situation does not occur, in general, in both B and D sectors because Λ_2^2 and Λ_{D2}^2 are not independent from each other due to a relation similar to Eq.(II.37). Conversely if we impose the two constraints (II.55), we can determine the pole masses Λ_2^2 and Λ_{D2}^2 . The relations (II.37) and (II.47) lead to a second order equation for Λ_{D2}^2 . However one root is negative and must be rejected. The second one gives Λ_{D2} in the $(2 - 3) \text{ GeV}$ range corresponding to Λ_2 in the $(5 - 6) \text{ GeV}$ range. The value of λ^B is not far from 1, the heavy quark symmetry prediction. Using as previously $m_b = 4.7 \text{ GeV}$ and $m_c = 1.45 \text{ GeV}$, we obtain $\Lambda_{D2} = 2.59 \text{ GeV}$, $\Lambda_2 = 5.93 \text{ GeV}$, $\lambda^B = 0.7698$ and $\lambda^D = 0.4068$.

9°) In the absence of a theoretical estimate for $a_-^{DK^*}(0)$ or $a_-^{BK^*}(0)$, we shall use in the B sector a model where λ^B is related to the pole mass Λ_2 by Eq.(II.55). The parameter λ^D is then computed from λ^B by inverting Eq.(II.47)

$$\lambda^D = \frac{\lambda^B - \sigma}{1 - \sigma \lambda^B} \quad (\text{II.56})$$

We plot in Figure 2 the ratios λ^B and λ^D as functions of Λ_2^2 , for values of Λ_2 in the range $(4 - 7) \text{ GeV}$. They are parts of hyperbolae with asymptotia parallel to the Λ_2^2 and λ axes. Eq.(II.37) which relates the pole masses in the B and D sectors is represented by a straight line in Figure 2. On the same Figure we also show, with dotted points, the quantity $\bar{\lambda}^D$ which corresponds to the second relation (II.55).

III Factorization and Kinematics, Experimental Data

1°) The two-body decays of the charged and neutral B mesons discussed in this paper are described, at the tree level, by the colour-suppressed diagram of Figure 3. Of course penguin diagrams also contribute to these decays at the one loop level. However the colourless charmonium states $\bar{c}c$ have to be excited from the vacuum and for which two or three gluons are needed. For that reason the penguin contribution will be neglected in this paper.

2^o) We consider the decay modes

$$B^+ \rightarrow K^+(K^{*+}) + \eta_c (J/\Psi, \Psi') \quad (\text{III.1})$$

$$B^0 \rightarrow K^0(K^{*0}) + \eta_c (J/\Psi, \Psi')$$

and we compute the decay amplitudes assuming factorization. We obtain an expression of the form

$$\langle \bar{c}c + \bar{s}q | T | \bar{b}q \rangle \propto \langle \bar{c}c | J^\mu | 0 \rangle \langle \bar{s}q | J_\mu | B \rangle \quad (\text{III.2})$$

The first factor in the right hand side of Eq.(III.2) involves the leptonic decay constants f_{η_c} , $f_{J/\Psi}$ and $f_{\Psi'}$ for η_c , J/Ψ and Ψ' respectively. The second factor is governed by the hadronic form factors for the $B \rightarrow K$ or $B \rightarrow K^*$ transitions. As a consequence, the branching ratios have the following structure

$$BR = BR_0 \cdot \left(\frac{f_{\bar{c}c}}{m_B} \right)^2 \cdot PS \cdot FF \quad (\text{III.3})$$

The scale BR_0 contains the Fermi coupling constant G_F , the Cabibbo-Kobayashi-Maskawa (CKM) factors $V_{cb}^* V_{cs}$ as indicated on Figure 3, the B meson life time τ_B and the BSW phenomenological constant a_2 describing colour-suppressed processes

$$BR_0 = \left[\frac{G_F m_B^2}{\sqrt{2}} \right]^2 |V_{cb}|^2 |V_{cs}|^2 \frac{m_B}{8\pi} a_2^2 \frac{\tau_B}{\hbar} \quad (\text{III.4})$$

Being interested only in ratios of decay widths, we shall not numerically compute BR_0 .

The quantity PS is a dimensionless phase space factor depending only on masses of the involved particles. Because of the small $K^+(K^{*+}) - K^0(K^{*0})$ mass differences, the numerical value of PS is slightly different for B^+ and B^0 decays. However the differences in these quantities are typically $\mathcal{O}(10^{-3})$, hence we ignore the mass difference between charged and neutral strange mesons. We give results for B^+ decay.

The last factor FF depends on the hadronic form factors and it contains the dynamics of the weak decays. The results are :

$$(a) \quad B^+ \rightarrow K^+ + \eta_c \quad : \quad PS = 0.3265 \quad , \quad FF = |F_0^{BK}(m_{\eta_c}^2)|^2 \quad (\text{III.5})$$

$$(b) \quad B^+ \rightarrow K^+ + J/\Psi \quad : \quad PS = 0.1296 \quad , \quad FF = |F_1^{BK}(m_{J/\Psi}^2)|^2 \quad (\text{III.6})$$

$$(c) \quad B^+ \rightarrow K^+ + \Psi' \quad : \quad PS = 0.0575 \quad , \quad FF = |F_1^{BK}(m_{\Psi'}^2)|^2 \quad (\text{III.7})$$

$$(d) \quad B^+ \rightarrow K^{*+} + \eta_c \quad : \quad PS = 0.1218 \quad , \quad FF = |A_0^{BK^*}(m_{\eta_c}^2)|^2 \quad (\text{III.8})$$

$$(e) \quad B^+ \rightarrow K^{*+} + J/\Psi \quad : \quad PS = 0.1399 \quad , \quad FF = |A_1^{BK^*}(m_{J/\Psi}^2)|^2 [(a - bx)^2 + 2(1 + c^2 y^2)] \quad (\text{III.9})$$

$$(f) \quad B^+ \rightarrow K^{*+} + \Psi' \quad : \quad PS = 0.1407 \quad , \quad FF = |A_1^{BK^*}(m_{\Psi'}^2)|^2 [(a' - b'x')^2 + 2(1 + c'^2 y'^2)] \quad (\text{III.10})$$

The analytic expressions for a, b, c are previously given in Ref.[1] and a', b', c' are obtained respectively from a, b, c by the simple substitution $m_{\Psi'}$ to $m_{J/\Psi}$. We get numerically :

$$a = 3.1652 \quad b = 1.3081 \quad c = 0.4356 \quad (\text{III.11})$$

$$a' = 2.0514 \quad b' = 0.5538 \quad c' = 0.3092 \quad (\text{III.12})$$

The ratios of form factors x, y, x', y' are defined by :

$$x \equiv x^B(m_{J/\Psi}^2) = \frac{A_2^{BK^*}(m_{J/\Psi}^2)}{A_1^{BK^*}(m_{J/\Psi}^2)} \quad , \quad y \equiv y^B(m_{J/\Psi}^2) = \frac{V^{BK^*}(m_{J/\Psi}^2)}{A_1^{BK^*}(m_{J/\Psi}^2)} \quad (\text{III.13})$$

$$x' \equiv x^B(m_{\Psi'}^2) = \frac{A_2^{BK^*}(m_{\Psi'}^2)}{A_1^{BK^*}(m_{\Psi'}^2)} \quad , \quad y' \equiv y^B(m_{\Psi'}^2) = \frac{V^{BK^*}(m_{\Psi'}^2)}{A_1^{BK^*}(m_{\Psi'}^2)} \quad (\text{III.14})$$

In the $K^* + J/\Psi$ and $K^* + \Psi'$ modes, we have three possible polarization states, one is longitudinal and two are transverse for both final particles. We shall define two interesting quantities, the fractional longitudinal polarization :

$$\rho_L = \frac{\Gamma(B \rightarrow K^* + J/\Psi)_{LL}}{\Gamma(B \rightarrow K^* + J/\Psi)} \quad , \quad \rho'_L = \frac{\Gamma(B \rightarrow K^* + \Psi')_{LL}}{\Gamma(B \rightarrow K^* + \Psi')} \quad (\text{III.15})$$

and the left-right asymmetry :

$$\mathcal{A}_{LR} = \frac{\Gamma(B \rightarrow K^* + J/\Psi)_{--} - \Gamma(B \rightarrow K^* + J/\Psi)_{++}}{\Gamma(B \rightarrow K^* + J/\Psi)_{--} + \Gamma(B \rightarrow K^* + J/\Psi)_{++}} \quad (\text{III.16})$$

$$\mathcal{A}'_{LR} = \frac{\Gamma(B \rightarrow K^* + \Psi')_{--} - \Gamma(B \rightarrow K^* + \Psi')_{++}}{\Gamma(B \rightarrow K^* + \Psi')_{--} + \Gamma(B \rightarrow K^* + \Psi')_{++}}$$

We get

$$\rho_L = \frac{(a - bx)^2}{(a - bx)^2 + 2[1 + c^2 y^2]} \quad , \quad \rho'_L = \frac{(a' - b'x')^2}{(a' - b'x')^2 + 2[1 + c'^2 y'^2]} \quad (\text{III.17})$$

$$\mathcal{A}_{LR} = \frac{2cy}{1 + c^2 y^2} \quad , \quad \mathcal{A}'_{LR} = \frac{2c'y'^2}{1 + c'^2 y'^2} \quad (\text{III.18})$$

We shall also define two ratios of rates

$$R_{J/\Psi} = \frac{\Gamma(B \rightarrow K^* + J/\Psi)}{\Gamma(B \rightarrow K + J/\Psi)} \quad , \quad R_{\Psi'} = \frac{\Gamma(B \rightarrow K^* + \Psi')}{\Gamma(B \rightarrow K + \Psi')} \quad (\text{III.19})$$

3^o) The experimental data for decay rates as averaged by PDG [5] are given in Table 1.

Modes	B^+	B^0
$K + \Psi'$	$(0.69 \pm 0.31) \cdot 10^{-3}$	$< 0.8 \cdot 10^{-3}$
$K^* + \Psi'$	$< 3.0 \cdot 10^{-3}$	$(1.4 \pm 0.9) \cdot 10^{-3}$
$K + J/\Psi$	$(1.02 \pm 0.14) \cdot 10^{-3}$	$(0.75 \pm 0.21) \cdot 10^{-3}$
$K^* + J/\Psi$	$(1.7 \pm 0.5) \cdot 10^{-3}$	$(1.58 \pm 0.28) \cdot 10^{-3}$

Table 1.

The modes $K\eta_c$ and $K^*\eta_c$ have not yet been observed experimentally.

The ratios $R_{\Psi'}$ and $R_{J/\Psi}$ can be estimated from the data of Table 1 and the results are shown in Table 2.

Ratios	B^+	B^0	B^+, B^0 combined
$R_{\Psi'}$	$< 4.35 \pm 1.95$	$> 1.75 \pm 1.12$	2.03 ± 1.59
$R_{J/\Psi}$	1.67 ± 0.54	2.11 ± 0.70	1.83 ± 0.43

Table 2.

A direct measurement of $R_{J/\Psi}$ by CLEO II [14] is consistent with our estimate given in the last column of Table 2,

$$\text{CLEO II} \quad R_{J/\Psi} = 1.71 \pm 0.34$$

In what follows we shall use the constraint $R_{J/\Psi} \leq 2.2$.

A second type of useful experimental data which turns out to be crucial is the fractional longitudinal polarization for the mode $B \rightarrow K^* + J/\Psi$ measured by 3 groups :

CLEO II [14]	$\rho_L = 0.80 \pm 0.08 \pm 0.05$
CDF [15]	$\rho_L = 0.66 \pm 0.10^{+0.10}_{-0.08}$
ARGUS [16]	$\rho_L = 0.97 \pm 0.16 \pm 0.15$

Averaging these results by the standard weighted least-squares procedure, we obtain :

$$\rho_L = 0.780 \pm 0.073$$

In what follows we shall use the one standard deviation lower limit $\rho_L \geq 0.7$.

IV The decay modes $B \rightarrow K(K^*) + \Psi'$

1°) We first consider the decay $B \rightarrow K^* + \Psi'$ which is described by the three hadronic form factors $A_1^{BK^*}$, $A_2^{BK^*}$ and V^{BK^*} taken at $q^2 = m_{\Psi'}^2$. In part II we have explained how the Isgur-Wise relations due to SU(2) heavy flavour symmetry allow one to compute the form factors in the B sector at $q^2 = t_B^{*o}$ from the values of the form factors in the D sector at $q^2 = 0$. It turns out that the numerical value of $t_B^{*o} = 13.4933 \text{ GeV}^2$, as given by Eq.(II.8), is remarkably close to $m_{\Psi'}^2 = 13.5866 \text{ GeV}^2$. Different plausible choices of m_b, m_c satisfying $m_b - m_c = 3.4 \pm 0.2 \text{ GeV}$ (as dictated by HQET scheme) yield similar results, t_B^{*o} is close to $m_{\Psi'}^2$. It is then justified to neglect the variation of the form factors between t_B^{*o} and $m_{\Psi'}^2$, and we obtain from Eqs.(II.31) and (II.32) :

$$A_1^{BK^*}(m_{\Psi'}^2) = 0.8056 \ C_{bc} \ A_1^{DK^*}(0) \quad (\text{IV.1})$$

$$V^{BK^*}(m_{\Psi'}^2) = 1.2413 \ C_{bc} \ V^{DK^*}(0) \quad (\text{IV.2})$$

With the values in the D sector as given by the Particle Data Group [5] (PDG henceforth) :

$$A_1^{DK^*}(0) = 0.56 \pm 0.04 \quad , \quad V^{DK^*}(0) = 1.1 \pm 0.2 \quad (\text{IV.3})$$

we get :

$$A_1^{BK^*}(m_{\Psi'}^2) = (0.4511 \pm 0.0322) \ C_{bc} \quad (\text{IV.4})$$

$$V^{BK^*}(m_{\Psi'}^2) = (1.3654 \pm 0.2483) \ C_{bc} \quad (\text{IV.5})$$

In an analogous way, we have :

$$y^B(m_{\Psi'}^2) = 1.5409 \ y^D(0) \quad (\text{IV.6})$$

and with the PDG values [5], $y^D(0) = 1.89 \pm 0.25$, we get

$$y' \equiv y^B(m_{\Psi'}^2) = 2.9123 \pm 0.3852 \quad (\text{IV.7})$$

The values of A_1^{BK*} and V^{BK*} at $q^2 = m_{\Psi'}^2$ are simply given by the Isgur-Wise relations from the values of A_1^{DK*} and V^{DK*} at $q^2 = 0$ and the relative error is the same in both B and D sectors.

For A_2^{BK*} and x^B the situation is not so simple because of the presence of the parameter λ^D ,

$$A_2^{BK*}(m_{\Psi'}^2) = [0.8121 + 0.4291 \lambda^D] C_{bc} A_2^{DK*}(0) \quad (\text{IV.8})$$

$$x' \equiv x^B(m_{\Psi'}^2) = [1.0081 + 0.5328 \lambda^D] x^D(0) \quad (\text{IV.9})$$

Using the model introduced in Part II, λ^B and λ^D are functions of the pole mass Λ_2 of the form factor A_2^{BK*} and represented on Figure 2. Using the PDG values [5] :

$$A_2^{DK*}(0) = 0.40 \pm 0.08 \quad , \quad x^D(0) = 0.73 \pm 0.15 \quad (\text{IV.10})$$

we obtain the quantities $A_2^{BK*}(m_{\Psi'}^2)$ and $x^B(m_{\Psi'}^2)$ as two functions of Λ_2 . They are represented on Figure 4 for Λ_2 in the range (5 - 6) GeV .

2°) The knowledge of $y' = y^B(m_{\Psi'}^2)$ determines the left-right asymmetry \mathcal{A}'_{LR} previously defined. The result

$$\mathcal{A}'_{LR} = 0.9945 \pm 0.0137 \quad (\text{IV.11})$$

shows that the dominant transverse amplitude has the helicity $\lambda = -1$. In the one standard deviation limit, $\mathcal{A}'_{LR} > 0.98$.

As a second consequence of the knowledge of y' , we can derive an upper bound for the longitudinal polarization ρ'_L :

$$\rho'_L \leq \frac{a'^2}{a'^2 + 2[1 + c'^2 y'^2]} \quad (\text{IV.12})$$

In the one standard deviation limit for y' , we obtain

$$\rho'_L \leq 0.5664 \quad (\text{IV.13})$$

which is significantly smaller than the theoretical upper bound $\rho'_L \leq a'^2/(a'^2 + 2) = 0.678$. These two results (IV.11) and (IV.13) are clearly scenario independent and they are direct

consequences of the Isgur-Wise SU(2) heavy flavour symmetry assuming the numerical value of $y^D(0)$ as given by experiment to be correct.

3^o) Furthermore not only the upper bound Eq.(IV.13) but also exact values for ρ'_L can be computed from $x' = x^B(m_{\Psi'}^2)$ and $y' = y^B(m_{\Psi'}^2)$. It is a function of Λ_2 represented on Figure 5 for Λ_2 in the (5 – 6) GeV range. The error on ρ'_L , $\Delta\rho'_L$, combines, in a quadratic way, the error on x' and y' due to those on $x^D(0)$ and $y^D(0)$. We observe that ρ'_L is a slowly increasing function of Λ_2 which takes the value $\rho'_L = 0.403 \pm 0.042$ for $\Lambda_2 = 6$ GeV. In Figure 5 we also represent our one standard deviation upper bound (IV.13).

Estimates for ρ'_L have been obtained by Kamal and Santra [9]. However their method and results are different from ours. Seven scenarios are considered for relating the J/Ψ and Ψ' modes and the allowed domains in the x', y' plane for each scenario are limited by the constraint $\rho_L \geq 0.68$. The upper bound for ρ'_L found in [9] is the theoretical upper bound $\rho'_{L, MAX} = a'^2/(a'^2 + 2) = 0.678$. Their lower bound is slightly scenario-dependent and it varies from 0.48 to 0.55. In our case, the allowed domain for ρ'_L , with one standard deviation, is smaller than their results and we have : $0.286 \leq \rho'_L \leq 0.445$.

Let us emphasize again that our prediction is scenario-independent. It is determined by using both the Isgur-Wise relation and the experimental values of $x^D(0)$, $y^D(0)$. It is interesting to compare the experimentally observed ρ_L with the theoretical prediction for ρ'_L :

$$\rho_L = 0.78 \pm 0.073 \quad , \quad 0.286 \leq \rho'_L \leq 0.445$$

4^o) Let us consider now the decay mode $B \rightarrow K + \Psi'$ described by the hadronic form factor $F_1^{BK}(m_{\Psi'}^2)$. The Isgur-Wise relation (II.19) gives $F_1^{BK}(t_B^o)$ in terms of $F_1^{DK}(0)$ and Λ_F via the parameter μ^D

$$F_1^{BK}(t_B^o) = [1.1779 + 0.6225 \mu^D] C_{bc} F_1^{DK}(0) \quad (\text{IV.14})$$

We use the PDG value [5] $F_1^{DK}(0) = 0.75 \pm 0.03$.

In order to obtain $F_1^{BK}(m_{\Psi'}^2)$, we must extrapolate F_1^{BK} from $t_B^o = 14.7288$ GeV² to $m_{\Psi'}^2 = 13.5866$. For that purpose we use, as explained in Part II, a monopole q^2 -dependence with a pole mass Λ_F . The result is shown on Figure 6 with Λ_F in the (5 – 6) GeV range. We observe that $F_1^{BK}(m_{\Psi'}^2)$ is a slowly decreasing function of Λ_F .

5^o) We finally study the ratio of rates

$$R_{\Psi'} = \frac{\Gamma(B \rightarrow K^* + \Psi')}{\Gamma(B \rightarrow K + \Psi')}$$

Using the phase space factors given in Part III, we obtain

$$R_{\Psi'} = 2.4455 \frac{(a' - b' x')^2 + 2[1 + c'^2 y'^2]}{z'^2} \quad (\text{IV.15})$$

where the ratio of form factors, z' , is defined by

$$z' = z^B(m_{\Psi'}^2) = \frac{F_1^{BK}(m_{\Psi'}^2)}{A_1^{BK^*}(m_{\Psi'}^2)} \quad (\text{IV.16})$$

The ratio z' , like $F_1^{BK}(m_{\Psi'}^2)$, is a function of Λ_F^2 and the ratio x' is a function of Λ_2^2 . It follows that $R_{\Psi'}$ depends on both parameters Λ_F^2 and Λ_2^2 . Restricting Λ_F and Λ_2 in the range (5 – 6) *GeV*, we find the ratio $R_{\Psi'}$ between 1.44 ± 0.28 (for $\Lambda_F = \Lambda_2 = 5$ *GeV*) and 2.92 ± 0.54 (for $\Lambda_F = \Lambda_2 = 6$ *GeV*).

Both extreme values are obviously compatible with the experimental estimate given in Part III :

$$(R_{\Psi'})_{exp} = 2.03 \pm 1.59 \quad (\text{IV.17})$$

The ratio of rates $R_{\Psi'}$ is represented on Figure 7 as a function of Λ_2 and Λ_F both in the range (5 – 6) *GeV*.

V The decay modes $B \rightarrow K(K^*) + J/\Psi$

1^o) We consider the decay mode $B \rightarrow K^* + J/\Psi$ described by the three form factors $A_1^{BK^*}(m_{J/\Psi}^2)$, $A_2^{BK^*}(m_{J/\Psi}^2)$ and $V^{BK^*}(m_{J/\Psi}^2)$. These quantities are related to the values at t_B^{*o} of the same form factors, already obtained in Part IV, via the scenario dependent parameters α_1 , α_2 and β .

$$\begin{aligned} A_1^{BK^*}(m_{J/\Psi}^2) &= \alpha_1 A_1^{BK^*}(t_B^{*o}) \\ A_2^{BK^*}(m_{J/\Psi}^2) &= \alpha_2 A_2^{BK^*}(t_B^{*o}) \\ V^{BK^*}(m_{J/\Psi}^2) &= \beta V^{BK^*}(t_B^{*o}) \end{aligned} \quad (\text{V.1})$$

and for their ratios

$$x \equiv x^B(m_{J/\Psi}^2) = p x^B(t_B^{*o}) \quad (\text{V.2})$$

$$y \equiv y^B(m_{J/\Psi}^2) = q y^B(t_B^{*o}) \quad (\text{V.3})$$

where

$$p = \frac{\alpha_2}{\alpha_1} \quad , \quad q = \frac{\beta}{\alpha_1} \quad (\text{V.4})$$

We shall consider the scenarios qualitatively described in the Part II. For that purpose, we introduce the function $r(\Lambda)$ defined by :

$$r(\Lambda) = \frac{\Lambda^2 - t_B^{*o}}{\Lambda^2 - m_{J/\Psi}^2} \quad (\text{V.5})$$

and we get :

$$\alpha_1 = [r(\Lambda_1)]^{n_1} \quad , \quad \alpha_2 = [r(\Lambda_2)]^{n_2} \quad , \quad \beta = [r(\Lambda_V)]^{n_V} \quad (\text{V.6})$$

For the algebraic integers n_1, n_2, n_V , we shall consider the four cases $n_i = -1, 0, +1, +2$ with $i = 1, 2, V$. On physical grounds, we impose to the pole masses $\Lambda_1, \Lambda_2, \Lambda_V$ to be inside a cube $5 \text{ GeV} \leq \Lambda_i \leq 6 \text{ GeV}$.

2°) We first study the scenario constraints due to the longitudinal polarization fraction ρ_L in $B \rightarrow K^* + J/\Psi$. For the $4^3 = 64$ possible triplets $[n_1, n_2, n_V]$, we compute ρ_L using the values of $x^B(t_B^{*o})$ and $y^B(t_B^{*o})$ obtained in Eqs.(IV.9) and (IV.7) and restricting the pole mass parameters Λ_1, Λ_2 and Λ_V inside the cube $5 \text{ GeV} \leq \Lambda_i \leq 6 \text{ GeV}$. We impose the experimental constraint in the form $\rho_L + \Delta\rho_L \geq 0.7$ where the error $\Delta\rho_L$ is computed in quadrature [See Appendix A-1] from the errors on x' and y' due to the experimental errors of $x^D(0)$ and $y^D(0)$. After numerically scanning over the whole n_i and Λ_i spaces, finally our results can be summarized in the following :

1. No solution is obtained when $n_1 = 0, 1, 2$ for the 16 values of the couple $[n_2, n_V]$.
2. Solutions exist only when $n_1 = -1$, i.e., when the form factor A_1 exhibits a linear decrease with q^2 . Of course, in this case, Λ_1 is no more a pole mass but simply a slope coefficient and it is reasonable now to relax the constraint $\Lambda_1 \leq 6 \text{ GeV}$ and to use only $\Lambda_1 \geq 5 \text{ GeV}$ in order to exclude a too fast variation with q^2 of A_1 .

3. We obtain solutions for only 4 triplets $[n_1, n_2, n_V]$:

$$[-1, 2, 2] \ ; \ [-1, 1, 2] \ ; \ [-1, 0, 2] \ ; \ [-1, 2, 1]$$

The allowed domains in the Λ_1 , Λ_2 , and Λ_V space are respectively represented on Figures 8, 9, 10 and 11.

4. In the four surviving triplets mentioned above, the maximal value of ρ_L occurs at $\Lambda_1 = 5 \text{ GeV}$, $\Lambda_2 = 6 \text{ GeV}$, $\Lambda_V = 5 \text{ GeV}$ and in the most favorable situation of two dipole q^2 dependence for A_2 and V , we obtain $\rho_L = 0.7162 \pm 0.0236$. Therefore $\rho_L = 0.74$ is the maximal value, within one standard deviation, we can get in our approach, considering only the quantity ρ_L .
5. It is interesting to notice that in the case of two monopole q^2 -dependence for A_2 and V , the maximal value of ρ_L is obtained at the point $\Lambda_1 = \Lambda_V = 5 \text{ GeV}$, $\Lambda_2 = 6 \text{ GeV}$ with the result $\rho_L = 0.6635 \pm 0.0339$, e.g., a one standard deviation value very close to 0.7. However we consider this case as only marginal, since, in any way, it will be eliminated when the second quantity $R_{J/\Psi}$ enters in the fit.

3^o) The second quantity is the ratio of rates $R_{J/\Psi}$ which has the form :

$$R_{J/\Psi} = 1.0793 \frac{(a - b x)^2 + 2(1 + c^2 y^2)}{z^2} \quad (\text{V.7})$$

where the ratio z is defined by :

$$z \equiv z^B(m_{J/\Psi}^2) = \frac{F_1^{BK}(m_{J/\Psi}^2)}{A_1^{BK*}(m_{J/\Psi}^2)} \quad (\text{V.8})$$

We impose the experimental constraint under the form $R_{J/\Psi} - \Delta R_{J/\Psi} \leq 2.2$ where the theoretical error $\Delta R_{J/\Psi}$ is computed, in quadrature [See Appendix A-3], from the experimental errors on $x^D(0)$, $y^D(0)$ and $z^D(0)$.

Our constraint $\rho_L + \Delta \rho_L \geq 0.7$ has selected four scenarios and the allowed domain at fixed Λ_V, Λ_2 is defined by :

$$\Lambda_1 \leq \Lambda_{1, \text{MAX}}(\Lambda_V, \Lambda_2) \quad (\text{V.9})$$

These domains have been represented on Figures 8 to 11 for $5 \text{ GeV} \leq \Lambda_V, \Lambda_2 \leq 6 \text{ GeV}$.

On the other hand, the constraint on $R_{J/\Psi}$ implies a lower limit for Λ_1

$$\Lambda_1 \geq \Lambda_{1, MIN}(\Lambda_V, \Lambda_2, \Lambda_F) \quad (\text{V.10})$$

Physical values of Λ_1 exist when and only when the lower limit (V.10) is smaller than the upper limit (V.9).

At fixed Λ_V, Λ_2 , the quantity $\Lambda_{1, MIN}$ is an increasing function of Λ_F and we shall restrict $\Lambda_F \geq 5 \text{ GeV}$. Therefore at fixed Λ_V, Λ_2 , the physical domain for Λ_1 is defined by

$$\Lambda_{1, MIN}(\Lambda_V, \Lambda_2, \Lambda_F = 5 \text{ GeV}) \leq \Lambda_1 \leq \Lambda_{1, MAX}(\Lambda_V, \Lambda_2) \quad (\text{V.11})$$

After a numerical scanning, we do not obtain solution satisfying the constraint $R_{J/\Psi} - \Delta R_{J/\Psi} \leq 2.2$ with the scenario $[n_1, n_2, n_V] = [-1, 2, 1]$. For the three remaining scenarios $[n_1, n_2, n_V] = [-1, 2, 2], [-1, 1, 2]$ and $[-1, 0, 2]$, the physical regions are then represented on Figures 12, 13 and 14.

At fixed Λ_V, Λ_2 , we have

$$5 \text{ GeV} \leq \Lambda_F(\Lambda_V, \Lambda_2) \leq \Lambda_{F, MAX}(\Lambda_V, \Lambda_2) \quad (\text{V.12})$$

where $\Lambda_{F, MAX}$ is determined by

$$\Lambda_{1, MIN}[\Lambda_V, \Lambda_2, \Lambda_{F, MAX}(\Lambda_V, \Lambda_2)] = \Lambda_{1, MAX}(\Lambda_V, \Lambda_2) \quad (\text{V.13})$$

The quantity $\Lambda_{F, MAX}(\Lambda_V, \Lambda_2)$ is represented on Figures 15, 16 and 17 for the three surviving scenarios.

4°) Comment and Illustration

Starting with 64 scenarios for the q^2 -dependence of the hadronic form factors $A_1, A_2, V, [n_1, n_2, n_V]$ with $n_i = -1, 0, 1, 2$, only survive 3 scenarios $[-1, n_2, 2]$ with $n_2 = 0, 1, 2$ for which it is possible to find a non empty domain in the parameter space $\Lambda_1, \Lambda_2, \Lambda_V, \Lambda_F$ such that both experimental constraints $\rho_L + \Delta\rho_L \geq 0.7$ and $R_{J/\Psi} - \Delta R_{J/\Psi} \leq 2.2$ are simultaneously satisfied. The hadronic form factors F_1^{BK} has been chosen with a monopole q^2 -dependence, $n_F = 1$, consistent with data in the D sector and the condition F_0^{BK} constant determines the parameter μ^B – and then μ^D – as a function of Λ_F . A similar relation determines λ^B – and then λ^D – as a function of Λ_2 .

$\Lambda_1(GeV)$	ρ_L	$\rho_L + \triangle\rho_L$	$R_{J/\Psi}$	$R_{J/\Psi} - \triangle R_{J/\Psi}$
$n_2 = 2$				
$\Lambda_{1, MAX} = 8.112$	0.665 ± 0.035	0.700	2.089 ± 0.508	1.581
$\Lambda_1 = 6.810$	0.675 ± 0.033	0.708	2.263 ± 0.422	1.841
$\Lambda_{1, MIN} = 5.426$	0.694 ± 0.028	0.722	2.694 ± 0.494	2.200
$n_2 = 1$				
$\Lambda_{1, MAX} = 6.113$	0.663 ± 0.037	0.700	2.324 ± 0.524	1.800
$\Lambda_1 = 5.770$	0.671 ± 0.035	0.705	2.482 ± 0.475	2.007
$\Lambda_{1, MIN} = 5.426$	0.681 ± 0.033	0.714	2.714 ± 0.514	2.200
$n_2 = 0$				
$\Lambda_{1, MAX} = 5.292$	0.660 ± 0.040	0.700	2.625 ± 0.542	2.083
$\Lambda_1 = 5.237$	0.663 ± 0.039	0.702	2.680 ± 0.529	2.151
$\Lambda_{1, MIN} = 5.183$	0.665 ± 0.038	0.703	2.739 ± 0.539	2.200

Table 3.

The allowed domains for $\Lambda_1, \Lambda_2, \Lambda_V$ and Λ_F have been represented in 3 dimensional plots on Figures 12 to 17. However it might be useful to produce some numerical values obtained for ρ_L and $R_{J/\Psi}$ in these domains and for that purpose. we have choosen, as an illustration, $\Lambda_2 = 6 \text{ GeV}$, $\Lambda_V = \Lambda_F = 5 \text{ GeV}$, and for Λ_1 , three values, $\Lambda_{1, MAX}$, $\Lambda_{1, MIN}$ and an intermediate value between the extremes. The results are represented in Table 3. A glance at Table 3 shows how difficult it is to fit simultaneously the large ρ_L and the relatively small $R_{J/\Psi}$, their opposite trends making the fit so difficult have been equally noticed [2].

The relative error on $R_{J/\Psi}$ is larger than the one of ρ_L , and this feature is very useful for obtaining a fit. It is essentially due to the fact that $R_{J/\Psi}$, in addition to the errors on $x^D(0)$ and $y^D(0)$ that also enter in $\triangle\rho_L$, has an uncertainty on $z^D(0)$ which is important. While the relative error on ρ_L is between 4% and 6%, the one of $R_{J/\Psi}$ is between 18% and 24%.

From these numerical results and those illustrated on Figures 12 to 17, it is clear that the scenario with a dipole form factor A_2 is the one with the largest phase space in $\Lambda_1, \Lambda_2, \Lambda_V, \Lambda_F$.

In this case it is relatively easy to accommodate both ρ_L and $R_{J/\Psi}$. We notice that the largest value of ρ_L we can obtain in this model is $\rho_L = 0.694 \pm 0.028$ and for $R_{J/\Psi}$ the smallest value is 2.089 ± 0.508 .

For the scenario with a monopole form factor A_2 , the situation is less comfortable even if it is possible to fit data on ρ_L and $R_{J/\Psi}$. Now the quantity ρ_L varies by less than 3% and $R_{J/\Psi}$ by about 17%.

In the third case of a constant form factor A_2 , the allowed domain for the parameters $\Lambda_1, \Lambda_2, \Lambda_V, \Lambda_F$ is very small and this possibility, even if both ρ_L and $R_{J/\Psi}$ satisfy the constraints, appears to be very marginal, the values of ρ_L and $R_{J/\Psi}$ being at the limits of the constraints.

5°) The left-right asymmetry \mathcal{A}_{LR} for transverse polarization has not been experimentally measured. It has been defined in Eq.(III.18) and it depends only on the ratio $y = y^B(m_{J/\Psi}^2)$ unambiguously related in our model to $y^D(0)$. For our three selected models, we have

$$y^B(m_{J/\Psi}^2) = \frac{(\Lambda_V^2 - t_B^{*o})^2 (\Lambda_1^2 - t_B^{*o})}{(\Lambda_V^2 - m_{J/\Psi}^2)^2 (\Lambda_1^2 - m_{J/\Psi}^2)} \frac{m_c}{m_b} \left(\frac{m_B + m_{K^*}}{m_D + m_{K^*}} \right)^2 y^D(0) \quad (\text{V.14})$$

The difference between the scenarios $n_2 = 2, 1, 0$ is an allowed domain for Λ_1, Λ_2 and Λ_V illustrated on Figures 12, 13 and 14. Scanning inside these domains, we make the predictions

$$n_2 = 2 : \quad 0.867 < \mathcal{A}_{LR} < 0.945 \quad (\text{V.15})$$

$$n_2 = 1 : \quad 0.837 < \mathcal{A}_{LR} < 0.910 \quad (\text{V.16})$$

$$n_2 = 0 : \quad 0.837 < \mathcal{A}_{LR} < 0.856 \quad (\text{V.17})$$

The left-right asymmetry is large in the three selected cases, not as large as in the $K^* + \Psi'$ case where it is close to one. We observe that the differences between the three scenarios are moderate.

VI The decay modes $B \rightarrow K(K^*) + \eta_c$

1°) We start by considering the decay mode $B \rightarrow K + \eta_c$ governed by the form factor $F_0^{BK}(m_{\eta_c}^2)$. The Isgur-Wise relations determine $F_0^{BK}(t_B^o)$ from $F_0^{DK}(0)$ in a μ^D dependent way :

$$F_0^{BK}(t_B^o) = C_{bc} [0.8460 - 0.0057 \mu^D] F_0^{DK}(0) \quad (\text{VI.1})$$

We notice that the coefficient of μ^D is very small as compared with the μ^D -independent term in the bracket of Eq.(VI.1). It follows that $F_0^{BK}(t_B^o)$ only weakly depends on μ^D .

In Part II, we have choosen a monopole form factor for $F_1^{BK}(q^2)$ and the parameter μ^B has been related to the pole mass Λ_F in such a way to obtain a form factor F_0^{BK} independent on q^2 . From the previous considerations, the constant value of F_0^{BK} will be a weakly dependent function of Λ_F . With Λ_F in the range (5 - 6) *GeV*, F_0^{BK} increases slowly from 0.6286 ± 0.0251 (for $\Lambda_F = 5$ *GeV*) to 0.6327 ± 0.0253 (for $\Lambda_F = 6$ *GeV*). The quantity $F_1^{BK}(0) = F_0^{BK}(0)$ has been represented on Figure 6.

2^o) We now consider the decay mode $B \rightarrow K^* + \eta_c$ which is described by the hadronic form factors $A_0^{BK^*}(m_{\eta_c}^2)$. The Isgur-Wise relation (II.49) gives $A_0^{BK^*}$ in terms of $A_1^{BK^*}$, $A_2^{BK^*}$ and the parameter λ^B . With the choice made in Part II for λ^B

$$\lambda^B = \frac{m_B^2 - m_{K^*}^2}{\Lambda_2^2} \quad (\text{VI.2})$$

we get :

$$A_0^{BK^*}(m_{\eta_c}^2) = 3.4602 A_1^{BK^*}(m_{\eta_c}^2) - 2.4602 \left[1 - \frac{m_{\eta_c}^2}{\Lambda_2^2} \right] A_2^{BK^*}(m_{\eta_c}^2) \quad (\text{VI.3})$$

In Part V, we have obtained constraints on the scenarios allowed by the requirement to fit ρ_L and $R_{J/\Psi}$. They correspond to $n_1 = -1$ and $n_2 = 2, 1, 0$.

We introduce the function $S(\Lambda)$ in order to relate the values of the hadronic form factors at $q^2 = m_{\eta_c}^2$ to the values at $q^2 = t_B^{*o}$ where they are known from part II :

$$S(\Lambda) = \frac{\Lambda^2 - t_B^{*o}}{\Lambda^2 - m_{\eta_c}^2} \quad (\text{VI.4})$$

and we get :

$$A_0^{BK^*}(m_{\eta_c}^2) = \frac{3.4602}{S(\Lambda_1)} A_1^{BK^*}(t_B^{*o}) - 2.4602 \left[1 - \frac{m_{\eta_c}^2}{\Lambda_2^2} \right] [S(\Lambda_2)]^{n_2} A_2^{BK^*}(t_B^{*o}) \quad (\text{VI.5})$$

The form factor value $A_0^{BK^*}(m_{\eta_c}^2)$ is scenario-dependent, firstly by the value of n_2 ; $n_2 = 2, 1, 0$, secondly by the values of the pole masses Λ_1 and Λ_2 in the restricted domains described in Part V. We vary Λ_1 and Λ_2 inside these domains and obtain :

$$n_2 = 2 : \quad A_0^{BK^*}(m_{\eta_c}^2) = 1.3768 \pm 0.2928 \quad (\text{VI.6})$$

$$n_2 = 1 : \quad A_0^{BK^*}(m_{\eta_c}^2) = 1.3979 \pm 0.2677 \quad (\text{VI.7})$$

$$n_2 = 0 : \quad A_0^{BK^*}(m_{\eta_c}^2) = 1.3611 \pm 0.1790 \quad (\text{VI.8})$$

We observe that the results for $A_0^{BK^*}(m_{\eta_c}^2)$ is only weakly scenario-dependent due to cancellations between the two terms of Eq.(VI.5)

3^o) The ratio R_{η_c} of these two decay modes

$$R_{\eta_c} = \frac{\Gamma(B \rightarrow K^* + \eta_c)}{\Gamma(B \rightarrow K + \eta_c)} \quad (\text{VI.9})$$

is given by :

$$R_{\eta_c} = 0.3732 \left| \frac{A_0^{BK^*}(m_{\eta_c}^2)}{F_0^{BK}(m_{\eta_c}^2)} \right|^2 \quad (\text{VI.10})$$

Using the values (VI.6), (VI.7), (VI.8) for $A_0^{BK^*}(m_{\eta_c}^2)$, and Eq.(VI.1) for $F_0^{BK}(m_{\eta_c}^2)$, we obtain :

$$n_2 = 2 : \quad R_{\eta_c} = 1.7903 \pm 0.7748 \quad (\text{VI.11})$$

$$n_2 = 1 : \quad R_{\eta_c} = 1.8456 \pm 0.7221 \quad (\text{VI.12})$$

$$n_2 = 0 : \quad R_{\eta_c} = 1.7497 \pm 0.4810 \quad (\text{VI.13})$$

Taking these results all together, we obtain the one standard deviation bounds

$$1.02 \leq R_{\eta_c} \leq 2.56 \quad (\text{VI.14})$$

We comment, in the next Part VII, on the difference between these results and the bounds on R_{η_c} previously obtained in Ref.[10].

VII Comparison of Different Charmonium States Production

1^o) Ratio of decay widths with the same strange meson, K or K^* , and different charmonium states are interesting quantities involving the leptonic decay constant $f_{\bar{c}c}$. We define four such ratios referred to the most accurately measured J/Ψ production :

$$S = \frac{\Gamma(B \rightarrow K + \Psi')}{\Gamma(B \rightarrow K + J/\Psi)} \quad , \quad S^* = \frac{\Gamma(B \rightarrow K^* + \Psi')}{\Gamma(B \rightarrow K^* + J/\Psi)} \quad (\text{VII.1})$$

$$T = \frac{\Gamma(B \rightarrow K + \eta_c)}{\Gamma(B \rightarrow K + J/\Psi)} \quad , \quad T^* = \frac{\Gamma(B \rightarrow K^* + \eta_c)}{\Gamma(B \rightarrow K^* + J/\Psi)} \quad (\text{VII.2})$$

Assuming factorization and using the phase space estimates given in Part III, we obtain :

$$S = 0.4438 \left(\frac{f_{\Psi'}}{f_{J/\Psi}} \right)^2 \left| \frac{F_1^{BK}(m_{\Psi'}^2)}{F_1^{BK}(m_{J/\Psi}^2)} \right|^2 \quad (\text{VII.3})$$

$$S^* = 1.0057 \left(\frac{f_{\Psi'}}{f_{J/\Psi}} \right)^2 \left| \frac{A_1^{BK^*}(m_{\Psi'}^2)}{A_1^{BK^*}(m_{J/\Psi}^2)} \right|^2 \frac{(a' - b'x')^2 + 2(1 + c'^2 y'^2)}{(a - bx)^2 + 2(1 + c^2 y^2)} \quad (\text{VII.4})$$

$$T = 2.5180 \left(\frac{f_{\eta_c}}{f_{J/\Psi}} \right)^2 \left| \frac{F_0^{BK}(m_{\eta_c}^2)}{F_1^{BK}(m_{J/\Psi}^2)} \right|^2 \quad (\text{VII.5})$$

$$T^* = 0.8706 \left(\frac{f_{\eta_c}}{f_{J/\Psi}} \right)^2 \left| \frac{A_0^{BK^*}(m_{\Psi'}^2)}{A_1^{BK^*}(m_{J/\Psi}^2)} \right|^2 \frac{1}{(a - bx)^2 + 2(1 + c^2 y^2)} \quad (\text{VII.6})$$

The η_c modes have not been experimentally observed and the only available information refers to J/Ψ and Ψ' modes. Using the PDG data [5] collected in Table 1 of Part III, we obtain for S and S^* values given in Table 4 :

Ratio	B^+	B^0	B^+, B^0 combined
S	0.68 ± 0.32	$< 1.07 \pm 0.30$	0.68 ± 0.32
S^*	$< 1.76 \pm 0.52$	0.89 ± 0.59	0.89 ± 0.59

Table 4

2^o) Using the values [11] of $f_{J/\Psi} = (384 \pm 14) \text{ MeV}$ and $f_{\Psi'} = (282 \pm 14) \text{ MeV}$ as estimated from the decays $J/\Psi \rightarrow e^+e^-$ and $\Psi' \rightarrow e^+e^-$, we obtain :

$$\left(\frac{f_{\Psi'}}{f_{J/\Psi}} \right)^2 = 0.539 \pm 0.066 \quad (\text{VII.7})$$

and the quantity S is written :

$$S = [0.2392 \pm 0.0292] \left| \frac{F_1^{BK}(m_{\Psi'}^2)}{F_1^{BK}(m_{J/\Psi}^2)} \right|^2 \quad (\text{VII.8})$$

In our model, the hadronic form factor $F_1^{BK}(q^2)$ has a monopole q^2 dependence with a pole mass Λ_F and we simply have :

$$\left| \frac{F_1^{BK}(m_{\Psi'}^2)}{F_1^{BK}(m_{J/\Psi}^2)} \right|^2 = \left| \frac{\Lambda_F^2 - m_{J/\Psi}^2}{\Lambda_F^2 - m_{\Psi'}^2} \right|^2 \quad (\text{VII.9})$$

This ratio of form factors is a decreasing function of Λ_F^2 and so is the ratio S . At $\Lambda_F = 5 \text{ GeV}$, the prediction for S is :

$$S(\Lambda_F = 5 \text{ GeV}) = 0.4363 \pm 0.0537 \quad (\text{VII.10})$$

This prediction is in agreement, within one standard deviation, with the experimental value estimated in Table 2, $S_{exp} = 0.68 \pm 0.32$. Such an agreement continues to occur for larger values of Λ_F up to 6.27 GeV .

The range of Λ_F depends on the three scenarios corresponding to $n_2 = 2, 1, 0$ and they are deduced from Figures 15, 16, 17 respectively. We get :

$$n_2 = 2 : \quad 0.4363 \pm 0.0537 \geq S \geq 0.3505 \pm 0.0432 \quad (\text{VII.11})$$

$$n_2 = 1 : \quad 0.4363 \pm 0.0537 \geq S \geq 0.3790 \pm 0.0467 \quad (\text{VII.12})$$

$$n_2 = 0 : \quad 0.4363 \pm 0.0537 \geq S \geq 0.4181 \pm 0.0515 \quad (\text{VII.13})$$

The errors quoted in Eq.(VII.11), (VII.12) and (VII.13) are due to the uncertainty on the leptonic decay constants $f_{\Psi'}$ and $f_{J/\Psi}$. In conclusion, the theoretical predictions of our model for the three scenarios agree with experimental results within one standard deviation.

3^o) The analysis of the second ratio S^* is more complex because of a large number of form factors involved. Using Eq.(VII.7), we get :

$$S^* = [0.5424 \pm 0.0664] \left| \frac{A_1^{BK*}(m_{\Psi'}^2)}{A_1^{BK*}(m_{J/\Psi}^2)} \right|^2 \frac{(a' - b'x')^2 + 2(1 + c'^2 y'^2)}{(a - bx)^2 + 2(1 + c^2 y^2)} \quad (\text{VII.14})$$

In our model the form factor $A_1^{BK*}(q^2)$ is linearly decreasing with a slope Λ_1 , and we simply have

$$\frac{A_1^{BK*}(m_{\Psi'}^2)}{A_1^{BK*}(m_{J/\Psi}^2)} = \frac{\Lambda_1^2 - m_{\Psi'}^2}{\Lambda_1^2 - m_{J/\Psi}^2} \quad (\text{VII.15})$$

We have computed the ratio S^* for the three scenarios $n_2 = 2, 1, 0$ using the values of Λ_1 , Λ_2 and Λ_V inside the allowed domains obtained in Part V and represented respectively on Figures 12, 13 and 14.

The results of this scanning are :

$$n_2 = 2 : \quad 0.3287 \pm 0.0028 \leq S^* \leq 0.4135 \pm 0.0038 \quad (\text{VII.16})$$

$$n_2 = 1 : \quad 0.3489 \pm 0.0034 \leq S^* \leq 0.4015 \pm 0.0039 \quad (\text{VII.17})$$

$$n_2 = 0 : \quad 0.3763 \pm 0.0039 \leq S^* \leq 0.3867 \pm 0.0040 \quad (\text{VII.18})$$

The errors quoted in Eqs.(VII.16), (VII.17) and (VII.18) are computed in quadrature from those on the ratios $f_{\Psi'}/f_{J/\Psi}$, $x^D(0)$ and $y^D(0)$. The theoretical predictions of our model for the three scenarios agree, within one standard deviation, with the experimental results estimated in Table 2 : $S_{exp}^* = 0.89 \pm 0.59$.

4°) Kamal and Santra [9] have studied the ratios S and S^* denoted by them respectively as $1/R$ and $1/R'$. In the case of R , both monopole and dipole q^2 dependences for F_1^{BK} are considered with a pole mass $\Lambda_F = 5.43 \text{ GeV}$. Their conclusion is that a dipole behaviour for F_1^{BK} is needed in order to obtain an agreement for R between theory and experiment in the one standard deviation limit.

The apparent contradiction between our result (monopole for F_1^{BK}) and the one of Ref.[9] is essentially due to the large experimental error of 47% for the quantity S or R . With $\delta = 0.47$ the relation at first order in δ , $(1 \pm \delta)^{-1} = 1 \mp \delta$ is not valid and one standard deviation limit for S and one standard deviation limit for R are different concepts. However, since the main part of the experimental error is due to the $K + \Psi'$ mode and for that reason the consideration of one standard deviation for S (where $K + \Psi'$ enters in the numerator) seems to be more relevant than for R .

A similar situation occurs for S^* and R' . Here the experimental error is even larger, 66.7%, and it is mainly due to the $K^* + \Psi'$ mode which enters in the numerator of S^* . Again the one standard deviation limit for S^* and the one standard deviation limit for R' are different quantities.

Also the pole masses in Ref.[9] are taken only at some fixed values, while in our approach these poles sweep inside the allowed domains of Figs. 12 – 17.

For the ratio R' as previously done for ρ_L' , they propose seven scenarios. Furthermore, considering only in the one standard deviation limit for R' , they exclude four scenarios where A_1^{BK*} is either constant or linearly decreasing with q^2 and conclude that if factorization assumption were to be held, then the only scenarios that are consistent with experiment are those in which A_1^{BK*} rises with q^2 . We observe however that R' (or S^*) is not an independent ratio but related to the other ratios by $S^* R_{J/\Psi} = S R_{\Psi'}$, such that considering R' (or S^*) alone might be inadequate.

5°) Comparing the $K + \eta_c$ and $K + J/\Psi$ decay modes, we now consider the ratio T which de-

depends on the ratio of the decay constants f_{η_c} and $f_{J/\Psi}$. Unfortunately f_{η_c} is not experimentally known and we use theoretical estimates if we want to make predictions.

Rewriting Eq(VII.5) in the form :

$$T = 2.5180 \left(\frac{f_{\eta_c}}{f_{J/\Psi}} \right)^2 S_V(\Lambda_F) \quad (\text{VII.19})$$

where

$$S_V(\Lambda_F) = \left| \frac{F_0^{BK}(m_{\eta_c}^2)}{F_1^{BK}(m_{J/\Psi}^2)} \right|^2 \quad (\text{VII.20})$$

We compute S_V in our model where F_0^{BK} is constant and F_1^{BK} has a monopole q^2 dependence with the pole mass Λ_F . As a consequence, we simply have

$$S_V(\Lambda_F) = \left(1 - \frac{m_{J/\Psi}^2}{\Lambda_F^2} \right)^2 \quad (\text{VII.21})$$

The function S_V is an increasing function of Λ_F . The allowed values of Λ_F have been discussed in Part V (Figs. 15 – 17), and we obtain :

$$n_2 = 2 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.71 \text{ GeV} \quad , \quad 0.38 \leq S_V \leq 0.50 \quad (\text{VII.22})$$

$$n_2 = 1 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.39 \text{ GeV} \quad , \quad 0.38 \leq S_V \leq 0.45 \quad (\text{VII.23})$$

$$n_2 = 0 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.10 \text{ GeV} \quad , \quad 0.38 \leq S_V \leq 0.40 \quad (\text{VII.24})$$

As pointed out in Ref.[17], a measurement of the ratio T will provide an opportunity to extract the scalar decay constant f_{η_c} from experiment. Unfortunately the decay mode $B \rightarrow K + \eta_c$ has not been experimentally observed.

There exist various theoretical ways to estimate both f_{η_c} and $f_{J/\Psi}$. Using the estimate quoted in Ref.[17],

$$\frac{f_{\eta_c}}{f_{J/\Psi}} = 0.993 \quad (\text{VII.25})$$

we make predictions for the ratio T :

$$n_2 = 2 : \quad 0.94 \leq T \leq 1.24 \quad (\text{VII.26})$$

$$n_2 = 1 : \quad 0.94 \leq T \leq 1.12 \quad (\text{VII.27})$$

$$n_2 = 0 : \quad 0.94 \leq T \leq 0.99 \quad (\text{VII.28})$$

6^o) We finally discuss the last ratio T^* which, under the factorization assumption, has the form given in Eq.(VII.6). We compute T^* for the three scenarios $n_2 = 2, 1, 0$, using the values of Λ_1, Λ_2 and Λ_V inside the allowed domains obtained in Part V. By writing

$$T^* = \left(\frac{f_{\eta_c}}{f_{J/\Psi}} \right)^2 \bar{T}^* \quad (\text{VII.29})$$

we obtain

$$n_2 = 2 : \quad 0.6148 \pm 0.1108 \leq \bar{T}^* \leq 0.7740 \pm 0.1002 \quad (\text{VII.30})$$

$$n_2 = 1 : \quad 0.6097 \pm 0.1259 \leq \bar{T}^* \leq 0.7375 \pm 0.1347 \quad (\text{VII.31})$$

$$n_2 = 0 : \quad 0.6015 \pm 0.1415 \leq \bar{T}^* \leq 0.6217 \pm 0.1444 \quad (\text{VII.32})$$

If one accept the value (VII.25) for the ratio of leptonic constants, we make predictions for the ratio T^* which are weakly scenario-dependent :

$$n_2 = 2 : \quad 0.50 \leq T^* \leq 0.86 \quad (\text{VII.33})$$

$$n_2 = 1 : \quad 0.48 \leq T^* \leq 0.86 \quad (\text{VII.34})$$

$$n_2 = 0 : \quad 0.45 \leq T^* \leq 0.76 \quad (\text{VII.35})$$

7^o) These results are now compared with those obtained in a previous paper [10]. The scenario-dependent parameter S_V defined by Eq.(VII.20) is obviously very different from its value in the BSW model considered in Ref.[10]. The second scenario dependent parameter S_A entering in T^* :

$$\left| \frac{A_0^{BK^*}(m_{\eta_c}^2)}{A_1^{BK^*}(m_{J/\Psi}^2)} \right|^2 \equiv \left| \frac{A_0^{BK^*}(0)}{A_1^{BK^*}(0)} \right|^2 S_A \quad (\text{VII.36})$$

is computed in our model with the result :

$$n_2 = 2 : \quad 0.767 \leq S_A \leq 0.793 \quad (\text{VII.37})$$

$$n_2 = 1 : \quad 0.832 \leq S_A \leq 0.849 \quad (\text{VII.38})$$

$$n_2 = 0 : \quad 0.966 \leq S_A \leq 0.978 \quad (\text{VII.39})$$

For the BSW model [8], S_A is 1.069 .

The ratio S_A/S_V entering in the ratio R_{η_c} is more dependent on the scenario than separately S_V and S_A . The results in our model are :

$$n_2 = 2 : \quad 1.53 \leq S_A/S_V \leq 2.09 \quad (\text{VII.40})$$

$$n_2 = 1 : \quad 1.85 \leq S_A/S_V \leq 2.19 \quad (\text{VII.41})$$

$$n_2 = 0 : \quad 2.44 \leq S_A/S_V \leq 2.54 \quad (\text{VII.42})$$

The bounds used in our previous paper [10], $1 \leq S_A/S_V \leq 1.4$ are largely underestimated, essentially because of the behaviour in q^2 of the hadronic form factors : F_0^{BK} constant and $A_1^{BK^*}$ linearly decreasing with q^2 . Therefore, the predictions of Ref.[10] for R_{η_c} are smaller than those obtained here.

VIII Radiative Decay $B \rightarrow K^* + \gamma$

1°) The radiative decay $B \rightarrow K^* + \gamma$ does not occur at the tree level in the standard model. At the one loop level, we have the so-called Penguin diagrams and, for the case consider here, the dominant contribution is the one corresponding to the exchange of a virtual t quark. In this approximation, the branching ratio is given by :

$$BR(B \rightarrow K^* + \gamma) = \frac{\tau_B}{\hbar} \frac{\alpha G_F^2}{256 \pi^4} |V_{tb}|^2 |V_{ts}|^2 m_B^2 \left(1 - \frac{m_{K^*}^2}{m_B^2}\right)^3 |C_7(m_b)|^2 m_b^2 \{ \} \quad (\text{VIII.1})$$

where the quantity $\{ \}$ depends on the hadronic form factors associated to the tensor and pseudotensor current taken at $q^2 = 0$ and for real photon [18] :

$$\{ \} = \left(1 + \frac{m_s}{m_b}\right)^2 |v^T(0)|^2 + \left(1 - \frac{m_s}{m_b}\right)^2 |a_2^T(0)|^2 \quad (\text{VIII.2})$$

The quantity $C_7(m_b)$ is the Wilson coefficient associated to the relevant weak current. It takes into account the large QCD corrections [19] and it plays a determinant role in the numerical calculation of the rate.

We use the unitarity relation of the CKM matrix

$$V_{tb} V_{ts}^* = -V_{cb} V_{cs}^* - V_{ub} V_{us}^* \quad (\text{VIII.3})$$

and we neglect the second term in the right hand side of Eq.(VIII.3) being $\mathcal{O}(\sin^2\theta_c)$ with respect to the other ones.

Numerically we take

$$|V_{cs}| = 0.970 \quad , \quad |V_{cb}| = 0.040 \quad , \quad \tau_B = 1.54 \cdot 10^{-12} \text{ s} \quad , \quad m_b = 4.7 \text{ GeV} \quad (\text{VIII.4})$$

and for the Wilson coefficient, choosing $m_t = 174 \text{ GeV}$ and $\Lambda_{QCD} = 200 \text{ MeV}$, we have [19]
 $C_7(m_b) = 0.325$.

The result is

$$BR(B \rightarrow K^* + \gamma) = 4.45 \cdot 10^{-5} \quad \{ \quad \} \quad (\text{VIII.5})$$

The radiative decay mode $B \rightarrow K^* + \gamma$ has been experimentally observed by CLEO [20]

$$BR(B \rightarrow K^* + \gamma) = (4.5 \pm 1.5 \pm 0.9) \cdot 10^{-5} = (4.50 \pm 1.75) \cdot 10^{-5} \quad (\text{VIII.6})$$

and we get

$$\{ \quad \}_{exp} = 1.011 \pm 0.393 \quad (\text{VIII.7})$$

2°) Following Isgur and Wise [3], we assume that in the B meson at rest, the b quark spinor in the weak current satisfies the free Dirac equation for a spinor at rest : $\gamma_o b = b$. As a consequence, the tensor (pseudotensor) current is related to the vector (pseudovector) current

$$\bar{q} [\gamma_o, \gamma_j] b = -2 \bar{q} \gamma_j b \quad , \quad \bar{q} [\gamma_o, \gamma_j] \gamma_5 b = 2 \bar{q} \gamma_j \gamma_5 b \quad (\text{VIII.8})$$

It is then straight forward to compute the four tensor or pseudotensor hadronic form factors in terms of the usual BSW form factors V, A_1, A_2 and A_3 . The two relations of interest here are :

$$v^T(0) = a_2^T(0) = \left(1 + \frac{m_{K^*}}{m_B}\right) A_1^{BK^*}(0) + \left(1 - \frac{m_{K^*}}{m_B}\right) V^{BK^*}(0) \quad (\text{VIII.9})$$

In the limit $v^T(0) = a_2^T(0)$, the quantity $\{ \quad \}$ of Eq.(VIII.2) becomes in the approximation $m_s^2 \ll m_b^2$:

$$\{ \quad \} = 2|v^T(0)|^2 \quad (\text{VIII.10})$$

and using the estimate (VIII.7) for $\{ \quad \}$, we obtain

$$|v^T(0)|_{exp} = 0.71 \pm 0.14 \quad (\text{VIII.11})$$

The tensor and pseudotensor hadronic form factors have been computed in various models, quark constituents models [21], vector meson dominance models [22], lattice gauge theories

[23]. In most of the estimates, the equality $v^T(0) = a_2^T(0)$ is obtained with $v^T(0)$ in the range 0.5 to 1, e.g., consistent with the CLEO result (VIII.11). A recent estimate of Griffin, Masip and Mc Guigan using the Isgur-Wise relation gives $v^T(0) = 0.97 \pm 0.13$ [24].

It is amusing to observe that the BSW model [8] which fails [1] in explaining the ratio ρ_L and $R_{J/\Psi}$ in $B \rightarrow K(K^*) + J/\Psi$ produces for $v^T(0)$, using Eq.(VIII.9), a value $v^T(0) = 0.69$ in very nice agreement with experiment. *It is clear that the radiative decay $B \rightarrow K^* + \gamma$ is not a very efficient filter for models.*

3°) In our model, we compute $v^T(0) = a_2^T(0)$ using formule (VIII.9). For $A_1^{BK^*}$ we use a linearly decreasing function of q^2 and for V^{BK^*} a increasing function of q^2 of the dipole type, the input values being predicted at $q^2 = t_B^{*o}$ by the Isgur-Wise relations. As a consequence, we obtain a relatively large $A_1^{BK^*}(0)$ and a relatively small $V^{BK^*}(0)$. The minimal value of $v^T(0)$ corresponds, in the allowed domain of the parameter space, to $\Lambda_2 = 6 \text{ GeV}$, $\Lambda_V = 5 \text{ GeV}$, $\Lambda_1 = \Lambda_{1, MAX}(\Lambda_V, \Lambda_2)$ and the results are in the one standard deviation limit, including QCD corrections :

$$n_2 = 2 : \quad v^T(0) \geq 0.94 \quad (\text{VIII.12})$$

$$n_2 = 1 : \quad v^T(0) \geq 1.11 \quad (\text{VIII.13})$$

$$n_2 = 0 : \quad v^T(0) \geq 1.32 \quad (\text{VIII.14})$$

If the assumption $v^T(0) = a_2^T(0)$ is correct, we see that the scenario $n_2 = 2$ can accomodate the experimental result (VIII.11). A fit is clearly more difficult for the scenario $n_2 = 1$ and it seems to be impossible for the scenario $n_2 = 0$.

However we must be aware of the fact that the estimate of the QCD correction, which is scale dependent, may have some uncertainty which has been disregarded in the experimental error quoted in Eq.(VIII.11). A theoretical error has to be added which might be as large as 15 % [24].

IX $D \rightarrow \overline{K}(\overline{K}^*)$ Hadronic Form Factors

1°) The $B \rightarrow K(K^*)$ and $D \rightarrow K(K^*)$ hadronic form factors are related by the SU(2) heavy flavour symmetry of Isgur-Wise. From the considerations of Part II, it is clear that the q^2 -

dependence for the form factors F_1, A_1, A_2 and V , namely

- (i) Same values for n_1, n_2, n_V and n_F in both B and D sectors;
- (ii) The pole masses in these sectors are related by Eqs.(II.24) and (II.37) of Part II.

For F_0 and A_0 the situation might be different in the two sectors but the q^2 -dependence of F_0 is known from that of F_1 and for A_0 , its q^2 -dependence follows from that of A_1 and A_2 as explained in Part II.

2^o) Let us first consider the semi-leptonic decay of D mesons. The relevant hadronic form factors are F_1^{DK} for $D \rightarrow \overline{K} + \ell^+ + \nu_\ell$ and $A_1^{DK*}, A_2^{DK*}, V^{DK*}$ for $D \rightarrow \overline{K}^* + \ell^+ + \nu_\ell$. Using the dimensionless variable $t = q^2/m_D^2$, we introduce the dimensionless function $X(t)$:

$$X(t) = \frac{1}{\Gamma} \frac{d\Gamma}{dt} \quad (\text{IX.1})$$

which is independent of all parameters entering in the semi-leptonic relevant scale factor.

We recall that the quantities $x^D(0)$, $y^D(0)$ and $z^D(0)$ (used in this paper for normalizing the B sector) have been extracted from experimental data on semi-leptonic decay in a scenario-dependent way, because the variation with q^2 of the form factors $F_1^{DK}, A_1^{DK*}, A_2^{DK*}$ and V^{DK*} has not been measured.

3^o) The q^2 distribution in semi-leptonic decay $D \rightarrow \overline{K} + \ell^+ + \nu_\ell$ is written in terms of the hadronic form factor $F_1^{DK}(q^2)$ as

$$\frac{d\Gamma(D \rightarrow \overline{K} + \ell^+ + \nu_\ell)}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{cs}|^2 [K(q^2)]^3 |F_1^{DK}(q^2)|^2 \quad (\text{IX.2})$$

where the q^2 dependent momentum $K(q^2)$ is given by

$$K(q^2) = \frac{1}{2m_D} \left\{ (m_D^2 + m_K^2 - q^2)^2 - 4m_D^2 m_K^2 \right\}^{1/2} \quad (\text{IX.3})$$

In the zero lepton mass limit, $0 \leq q^2 \leq (m_D - m_K)^2$.

In our model $F_1^{DK}(q^2)$ has a monopole q^2 -dependence with a pole mass Λ_{DF} related to Λ_F in the B sector by Eq.(II.24). Defining the dimensionless parameters :

$$r = \frac{m_K}{m_D}, \quad \alpha_F = \frac{m_D^2}{\Lambda_{DF}^2} \quad (\text{IX.4})$$

we obtain for $X(t)$ the expression :

$$X(t) = \frac{1}{I(\alpha_F)} \frac{[(1 + r^2 - t)^2 - 4r^2]^{3/2}}{(1 - \alpha_F t)^2} \quad (\text{IX.5})$$

where the integral $I(\alpha_F)$ is defined by the normalization condition $X(t)$:

$$I(\alpha_F) = \int_0^{(1-r)^2} \frac{[(1+r^2-x)^2 - 4r^2]^{3/2}}{(1-\alpha_F x)^2} dx \quad (\text{IX.6})$$

Of course, the semi-leptonic rate is simply given by :

$$\Gamma(D \rightarrow \overline{K} + \ell^+ + \nu_\ell) = \frac{G_F^2 m_D^5}{192\pi^3} |V_{cs}|^2 |F_1^{DK}(0)|^2 I(\alpha_F) \quad (\text{IX.7})$$

The normalized distribution $X(t)$ for the semi-leptonic decay mode $D \rightarrow \overline{K} + \ell^+ + \nu_\ell$ is represented on Figure 18 for values of α_F corresponding to the bounds on Λ_F obtained in Part V and illustrated on Figures 15, 16 and 17.

$$n_2 = 2 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.71 \text{ GeV} \quad 1.097 \geq \alpha_F \geq 0.630 \quad (\text{IX.8})$$

$$n_2 = 1 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.39 \text{ GeV} \quad 1.097 \geq \alpha_F \geq 0.787 \quad (\text{IX.9})$$

$$n_2 = 0 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.10 \text{ GeV} \quad 1.097 \geq \alpha_F \geq 0.999 \quad (\text{IX.10})$$

The distribution $X(t)$ is a monotonically decreasing function of t . Its shape is not very sensitive to α_F except in the neighbourhood of $t = 0$.

An estimate for the slope of the q^2 distribution at $q^2 = 0$ has been given by Witherell [12] using two models for the q^2 -dependence of $F_1^{BK}(q^2)$, an exponential form and a monopole form. The result consistent for the two models translated in the α_F language is

$$0.76 \leq \alpha_F \leq 1.30 \quad (\text{IX.11})$$

which means in our model

$$5.36 \text{ GeV} \geq \Lambda_F \geq 5.02 \text{ GeV} \quad (\text{IX.12})$$

Therefore our bounds (IX.8) – (IX.10) are consistent with experiment.

4^o) The q^2 distribution in the semi-leptonic decay $D \rightarrow \overline{K}^* + \ell^+ + \nu_\ell$ depends on the three hadronic form factors $A_1^{DK^*}(q^2)$, $A_2^{DK^*}(q^2)$ and $V^{DK^*}(q^2)$. We have three possible polarizations for the final K^* , $\lambda = 0, \pm 1$.

It is convenient to define dimensionless parameters :

$$r^* = \frac{m_{K^*}}{m_D} \quad , \quad t = \frac{q^2}{m_D^2} \quad , \quad \alpha_j = \frac{m_D^2}{\Lambda_{Dj}^2} \quad ; \quad j = 1, 2, V \quad (\text{IX.13})$$

where the pole masses in the B and D sectors are related by Eq.(II.37).

The fixed q^2 distribution is given by :

$$\frac{d\Gamma(D \rightarrow \overline{K}^* + \ell^+ + \nu_\ell)_\lambda}{dt} = \frac{G_F^2 m_D^5}{192\pi^3} |V_{cs}|^2 (1 + r^*)^2 |A_1^{DK^*}(0)|^2 M_\lambda(t; \alpha_1, \alpha_2, \alpha_V) \quad (\text{IX.14})$$

where

$$M_L(t; \alpha_1, \alpha_2) = k(t) \left| \tilde{a}(t) \frac{1}{(1 - \alpha_1 t)^{n_1}} - \tilde{b}(t) \frac{x^D(0)}{(1 - \alpha_2 t)^{n_2}} \right|^2 \quad (\text{IX.15})$$

$$M_\pm(t; \alpha_1, \alpha_V) = t k(t) \left| \frac{1}{(1 - \alpha_1 t)^{n_1}} \mp \tilde{c}(t) \frac{y^D(0)}{(1 - \alpha_V t)^{n_V}} \right|^2 \quad (\text{IX.16})$$

with $k(t)$ given by :

$$k(t) = \left\{ (1 + r^{*2} - t)^2 - 4r^{*2} \right\}^{1/2} \quad (\text{IX.17})$$

The coefficients $\tilde{a}(t)$, $\tilde{b}(t)$ and $\tilde{c}(t)$ are :

$$\tilde{a}(t) = \frac{1 - r^{*2} - t}{2r^*} \quad (\text{IX.18})$$

$$\tilde{b}(t) = \frac{k^2(t)}{2r^*(1 + r^*)^2} \quad (\text{IX.19})$$

$$\tilde{c}(t) = \frac{k(t)}{(1 + r^*)^2} \quad (\text{IX.20})$$

We define the integrals :

$$I_\lambda(\alpha_1, \alpha_2, \alpha_V) = \int_0^{(1-r^*)^2} M_\lambda(t; \alpha_1, \alpha_2, \alpha_V) dt \quad (\text{IX.21})$$

and the integrated rate is given by :

$$\Gamma(D \rightarrow \overline{K}^* + \ell^+ + \nu_\ell) = \frac{G_F^2 m_D^5}{192\pi^3} |V_{cs}|^2 (1 + r^*)^2 |A_1^{DK^*}(0)|^2 \Sigma_\lambda I_\lambda(\alpha_1, \alpha_2, \alpha_V) \quad (\text{IX.22})$$

The normalized q^2 distribution is :

$$X(t) = \frac{\Sigma_\lambda M_\lambda(t; \alpha_1, \alpha_2, \alpha_V)}{\Sigma_\lambda I_\lambda(t; \alpha_1, \alpha_2, \alpha_V)} \quad (\text{IX.23})$$

Of course for the scenarios selected in Part V, we have $n_1 = -1, n_V = 2$ and three possible values for n_2 ; $n_2 = 2, 1, 0$. We have computed $X(t)$ in these three cases by using the PDG values [5] for $x^D(0)$ and $y^D(0)$. The parameters $\alpha_1, \alpha_2, \alpha_V$ – or equivalently $\Lambda_1, \Lambda_2, \Lambda_V$ – are constrained to stay inside the allowed domains represented on Figures 12, 13 and 14. The

results are shown on Figures 19, 20 and 21. As in the previous case, the largest sensitivity of $X(t)$ to the parameters α_i is in the neighbourhood of $t = 0$.

In an analogous way, it is possible to study the q^2 distributions for the polarization parameters $\rho_L^{sl}(t)$ and $\mathcal{A}_{LR}^{sl}(t)$:

$$\rho_L^{sl}(t) = \frac{M_L(t; \alpha_1, \alpha_2, \alpha_V)}{\Sigma_\lambda M_\lambda(t; \alpha_1, \alpha_2, \alpha_V)} \quad (\text{IX.24})$$

$$\mathcal{A}_{LR}^{sl}(t) = \frac{M_-(t; \alpha_1, \alpha_2, \alpha_V) - M_+(t; \alpha_1, \alpha_2, \alpha_V)}{M_-(t; \alpha_1, \alpha_2, \alpha_V) + M_+(t; \alpha_1, \alpha_2, \alpha_V)} \quad (\text{IX.25})$$

We only give here the integrated ratios ρ_L^{sl} and \mathcal{A}_{LR}^{sl} where the functions $M_\lambda(t; \alpha_1, \alpha_2, \alpha_V)$ in Eq.(IX.15) and (IX.16) are replaced by their integrals over t , $I_\lambda(\alpha_1, \alpha_2, \alpha_V)$. The results for the three cases $n_2 = 2, 1, 0$ are the following :

$$\begin{aligned} n_2 = 2 : \quad & 0.516 \leq \rho_L^{sl} \leq 0.541 \\ & 0.885 \geq \mathcal{A}_{LR}^{sl} \geq 0.829 \end{aligned} \quad (\text{IX.26})$$

$$\begin{aligned} n_2 = 1 : \quad & 0.526 \leq \rho_L^{sl} \leq 0.541 \\ & 0.904 \geq \mathcal{A}_{LR}^{sl} \geq 0.857 \end{aligned} \quad (\text{IX.27})$$

$$\begin{aligned} n_2 = 0 : \quad & 0.536 \leq \rho_L^{sl} \leq 0.538 \\ & 0.904 \geq \mathcal{A}_{LR}^{sl} \geq 0.892 \end{aligned} \quad (\text{IX.28})$$

In Eqs.(IX.26) - (IX.28) the results are presented in such a way to exhibit a correlation between the largest (smallest) ρ_L^{sl} and the smallest (largest) \mathcal{A}_{LR}^{sl} .

5^o) Let us now consider the hadronic two body decays $D^0 \rightarrow \overline{K}(K^{*-}) + \pi^+(\rho^+)$. Assuming factorization to be justified in the D sector, the various decay rates can be determined by using the hadronic $D \rightarrow \overline{K}(\overline{K}^*)$ form factors studied here.

At the quark level, the tree level diagram is of the spectator type and the various rates are written in the following way :

$$BR = BR_0 \cdot \left(\frac{f_{u\bar{d}}}{m_D} \right)^2 \cdot PS \cdot FF \quad (\text{IX.29})$$

The scale BR_0 is given by :

$$BR_0 = \left(\frac{G_F m_D^2}{\sqrt{2}} \right)^2 |V_{cs}|^2 |V_{ud}|^2 \frac{m_D}{8\pi} a_1^2 \frac{\tau_{D^0}}{\hbar} \quad (\text{IX.30})$$

where a_1 is the BSW [8] parameter for color favoured processes. As in the Part III, PS is a phase space factor, $f_{u\bar{d}}$ is a leptonic decay constant f_{π^+} or f_{ρ^+} which are experimentally known and FF depends on the hadronic form factors.

The results are :

$$a) \ D^0 \rightarrow K^- + \pi^+ \quad , \quad PS = 0.3993 \quad , \quad FF = |F_0^{DK}(m_\pi^2)|^2 \quad (\text{IX.31})$$

$$b) \ D^0 \rightarrow K^- + \rho^+ \quad , \quad PS = 0.1936 \quad , \quad FF = |F_1^{DK}(m_\rho^2)|^2 \quad (\text{IX.32})$$

$$c) \ D^0 \rightarrow K^{*-} + \pi^+ \quad , \quad PS = 0.2216 \quad , \quad FF = |A_0^{DK^*}(m_\pi^2)|^2 \quad (\text{IX.33})$$

$$d) \ D^0 \rightarrow K^{*-} + \rho^+ \quad , \quad PS = 0.0843$$

$$FF = |A_1^{DK^*}(m_\rho^2)|^2 \left\{ (a_D - b_D x_D)^2 + 2[1 + c_D^2 y_D^2] \right\} \quad (\text{IX.34})$$

where

$$a_D = 1.5310 \quad , \quad b_D = 0.2419 \quad , \quad c_D = 0.2087 \quad (\text{IX.35})$$

and

$$x_D = \frac{A_2^{DK^*}(m_\rho^2)}{A_1^{DK^*}(m_\rho^2)} \quad , \quad y_D = \frac{V^{DK^*}(m_\rho^2)}{A_1^{DK^*}(m_\rho^2)} \quad (\text{IX.36})$$

In order to test the assumption of factorization in the D sector, we shall study two ratios of rates :

$$R_\pi = \frac{\Gamma(D^0 \rightarrow K^{*-} + \pi^+)}{\Gamma(D^0 \rightarrow K^- + \pi^+)} \quad , \quad T_D = \frac{\Gamma(D^0 \rightarrow K^- + \rho^+)}{\Gamma(D^0 \rightarrow K^- + \pi^+)} \quad (\text{IX.37})$$

Using Eq.(IX.31) and (IX.33), we obtain

$$R_\pi = 0.5550 \left| \frac{A_0^{DK^*}(m_\pi^2)}{F_0^{DK}(m_\pi^2)} \right|^2 \quad (\text{IX.38})$$

It is legitimate to neglect the variation of the form factors F_0 and A_0 between $q^2 = 0$ and $q^2 = m_\pi^2$. Using the normalization conditions [8],

$$F_0^{DK}(0) = F_1^{DK}(0) \quad (\text{IX.39})$$

$$A_0^{DK^*}(0) = \frac{m_D + m_{K^*}}{2m_{K^*}} A_1^{DK^*}(0) - \frac{m_D - m_{K^*}}{2m_{K^*}} A_2^{DK^*}(0) \quad (\text{IX.40})$$

and the values at $q^2 = 0$ of the form factors as given by the PDG [5] :

$$F_1^{DK}(0) = 0.75 \pm 0.03 \quad , \quad A_1^{DK^*}(0) = 0.56 \pm 0.04 \quad , \quad A_2^{DK^*}(0) = 0.40 \pm 0.08 \quad (\text{IX.41})$$

we obtain :

$$A_0^{DK^*}(0) = 0.6473 \pm 0.0757 \quad (\text{IX.42})$$

and the theoretical prediction for R_π is

$$(R_\pi)_{th} = 0.4134 \pm 0.1022 \quad (\text{IX.43})$$

The experimental result is [5] :

$$(R_\pi)_{exp} = 1.22 \pm 0.16 \quad (\text{IX.44})$$

The discrepancy between theory and experiment is very large. The theoretical prediction is clearly scenario-independent and it is unlikely that final state interaction would be able to fill the gap between theory and experiment. The most probable explanation is a failure of factorization in the D sector.

Let us now consider the quantity T_D . Using Eq.(IX.31) and (IX.32), we get

$$T_D = 0.4850 \left(\frac{f_\rho}{f_\pi} \right)^2 \left| \frac{F_1^{DK}(m_\rho^2)}{F_0^{DK}(m_\pi^2)} \right|^2 \quad (\text{IX.45})$$

For the leptonic decay constants f_π and f_ρ , we use the experimental values

$$f_\pi = 131.7 \text{ MeV} \quad f_\rho = 212 \text{ MeV} \quad (\text{IX.46})$$

and T_D is written as :

$$T_D = 1.2566 \left| \frac{F_1^{DK}(m_\rho^2)}{F_0^{DK}(m_\pi^2)} \right|^2 \quad (\text{IX.47})$$

Neglecting as previously the variation of F_0 between $q^2 = 0$ and $q^2 = m_\pi^2$ and using the normalization condition (IX.39), we get

$$\frac{F_1^{DK}(m_\rho^2)}{F_0^{DK}(m_\pi^2)} \simeq \frac{F_1^{DK}(m_\rho^2)}{F_1^{DK}(0)} \quad (\text{IX.48})$$

In this paper we have used a monopole q^2 -dependence for F_1 , $n_F = 1$ and we have

$$\frac{F_1^{DK}(m_\rho^2)}{F_1^{DK}(0)} = \frac{1}{1 - \frac{m_\rho^2}{\Lambda_{DF}^2}} \quad (\text{IX.49})$$

where the pole masses Λ_F and Λ_{DF} in the B and D sectors are related by Eq.(II.24). With the constraint $\Lambda_F \geq 5 \text{ GeV}$, the maximal value of T_D is obtained with $\Lambda_{DF} = 1.78 \text{ GeV}$ – which corresponds to $\Lambda_F = 5 \text{ GeV}$ – with the result.

$$(T_D)_{th} < 1.8946 \quad (\text{IX.50})$$

The experimental value is significantly larger [5] :

$$(T_D)_{exp} = 2.59 \pm 0.34 \quad (\text{IX.51})$$

Again the factorization assumption seems to be in a bad shape in the D sector.

X Critical Discussions and Conclusions

1°) We have shown that the assumption of factorization is not ruled out by experimental data for the colour-suppressed decay modes of the B meson, $B \rightarrow K(K^*) + J/\Psi(\Psi')$. The failure pointed out in Ref.[1] might be due to inadequate choices of hadronic form factors and the aim of this paper is essentially to exhibit possible q^2 -dependences that are able to explain experimental data and particularly the ratios ρ_L and $R_{J/\Psi}$. Of course the possibility to understand experiment is not necessary a proof of factorization.

Let us first summarize the assumptions and constraints contained in our model.

(A) Assumptions :

1. Factorization holds for color suppressed B decays and final state strong interaction effects can be neglected.
2. The SU(2) heavy flavour symmetry between the b and c quarks is realized by the Isgur-Wise relations [3].
3. The input experimental data in the D sector are taken from the analysis of semi-leptonic decays $D \rightarrow \overline{K} + \ell^+ + \nu_\ell$ and $D \rightarrow \overline{K}^* + \ell^+ + \nu_\ell$ in the form of the values at $q^2 = 0$ of the $D \rightarrow K(K^*)$ hadronic form factors[5].

(B) The experimental constraints are :

1. The experimental rates for $B \rightarrow K + J/\Psi$, $B \rightarrow K^* + J/\Psi$, $B \rightarrow K + \Psi'$ and $B \rightarrow K^* + \Psi'$ used in the form of ratios of rates $R_{J/\Psi}$, $R_{\Psi'}$, S and S^* [defined respectively by Eq.(III.19) and Eq.(VII.1)].
2. The observed longitudinal polarization fraction ρ_L in $B \rightarrow K^* + J/\Psi$ [defined by Eq.(III.15)].

(C) The theoretical constraints are :

1. The explicit form of the q^2 dependence of the hadronic form factors F_1 , A_1 , A_2 , V choosen as $[1 - q^2/\Lambda^2]^{-n}$ with $n = -1, 0, 1, 2$.
2. The pole masses Λ of the various form factors in the B sector are limited to the (5–6) GeV range in order to relate them in a likely way to $b\bar{s}$ bound state masses.
3. The ratios of form factors $\mu^B(q^2)$ and $\lambda^B(q^2)$ defined in Eqs.(II.16) and (II.46) are assumed to be independent of q^2 and related in a natural way to the pole masses Λ_F and Λ_2 by Eqs.(II.26) and (II.55).

2^o) Among our three theoretical assumptions, the weakest one seems to us the third, i.e., the use of the experimental values at $q^2 = 0$ of the $D \rightarrow K(K^*)$ hadronic form factors as deduced in a scenario-dependent way from the experimental triple angular distribution of semi-leptonic decay. We must emphasize that measurements of the q^2 -dependence of these form factors are not available and their values at $q^2 = 0$ are obtained by extrapolation at $q^2 = 0$ of experimental data, assuming monopole q^2 -dependence for all form factors.

Consider first the simplest case of the form factor $F_1^{DK}(q^2)$. The two decay modes $D^+ \rightarrow \bar{K}^0 \ell^+ \nu_\ell$ and $D^0 \rightarrow K^- \ell^+ \nu_\ell$ are expected to have equal rates, the associated weak current $c\bar{s}$ being isoscalar. Experimentally these two rates differ by few standard deviations [5] and some average dominated by the most accurately measured D^0 mode is used for extracting the quantity $F_1^{DK}(0)$ and the slope of $F_1^{DK}(q^2)$ at $q^2 = 0$ from the measured q^2 distribution. In order to have an estimate of the uncertainties of the analysis, let us notice that the rate value used by Witherell [12] is 10 % higher than the one of the PDG [5] both values being given with 5 % errors. In spite of this difference, the values obtained by the PDG [5], $F_1^{DK}(0) = 0.75 \pm 0.03$ and by Witherell [12], $F_1^{DK}(0) = 0.77 \pm 0.04$ are very similar.

We have considered the problem of determining $F_1^{BK}(0)$ from the semi-leptonic rate by using Eq.(IX.7). The result depends on the parameter α_F simply related to the pole masses Λ_{DF} and Λ_F in the D and B sectors.

The results are :

$$n_2 = 2 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.71 \text{ GeV} \quad , \quad 0.68 \leq F_1^{DK}(0) \leq 0.79 \quad (\text{X.1})$$

$$n_2 = 1 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.39 \text{ GeV} \quad , \quad 0.68 \leq F_1^{DK}(0) \leq 0.77 \quad (\text{X.2})$$

$$n_2 = 0 : \quad 5 \text{ GeV} \leq \Lambda_F \leq 5.10 \text{ GeV} \quad , \quad 0.68 \leq F_1^{DK}(0) \leq 0.73 \quad (\text{X.3})$$

The numerical value of the rate used in this analysis is the PDG value of $(8.2 \pm 0.4) 10^{10} s^{-1}$. We notice that the slope results presented by Witherell [12] correspond in our language to $5.02 \text{ GeV} \leq \Lambda_F \leq 5.36 \text{ GeV}$.

We now comment on $D \rightarrow \overline{K}^* \ell^+ \nu_\ell$ data, for which the q^2 -dependences for the relevant hadronic form factors, $A_1^{K^*}$, $A_2^{DK^*}$ and V^{DK^*} , are not yet available. By fitting the q^2 distribution and the angular distribution due to the $K^* \rightarrow K + \pi$ decay, the ratios $x^D(0)$ and $y^D(0)$ are obtained assuming for the form factors a monopole q^2 -dependence with the pole masses given by the lowest lying $c\overline{s}$ bound states, $D_s^*(2110)$ and $D_s^{**}(2560)$. The results of the PDG [5] and of Witherell [12] are nearly identical. The quantity $A_1^{DK^*}(0)$ is then obtained from the transition rate. In this case the semi-leptonic decay rates $\Gamma(D^0 \rightarrow K^{*-} + \ell^+ + \nu_\ell)$ and $\Gamma(D^+ \rightarrow \overline{K}^{*0} + \ell^+ + \nu_\ell)$ are equal within errors. However the average value of the rate used by the PDG and by Witherell differs by 11% and as a consequence the values of $A_1^{DK^*}(0)$ obtained in the two cases are different :

$$PDG [5] \quad A_1^{DK^*}(0) = 0.56 \pm 0.04 \quad , \quad W [12] \quad A_1^{DK^*}(0) = 0.61 \pm 0.05 \quad (\text{X.4})$$

We study the problem of determining $A_1^{DK^*}(0)$ from the semi-leptonic decay rate by using Eq.(IX.22). We vary the parameters $\Lambda_1, \Lambda_2, \Lambda_V$ ($\alpha_1, \alpha_2, \alpha_V$) in the allowed domains determined in Part V and the results are :

$$n_2 = 2 : \quad 0.601 \leq A_1^{DK^*}(0) \leq 0.645 \quad (\text{X.5})$$

$$n_2 = 1 : \quad 0.618 \leq A_1^{DK^*}(0) \leq 0.645 \quad (\text{X.6})$$

$$n_2 = 0 : \quad 0.638 \leq A_1^{DK^*}(0) \leq 0.647 \quad (\text{X.7})$$

Our $A_1^{DK^*}(0)$ is to be compared to the PDG one 0.56 ± 0.04 , since the numerical value of the rate used in our analysis is the PDG value $(4.6 \pm 0.4) 10^{10} s^{-1}$.

The errors quoted for $F_1^{DK}(0)$ and $A_1^{DK^*}(0)$ in references [5] and [12] do not take into account the theoretical uncertainties due to the q^2 -dependence used for analysing the experimental data. The formal results previously obtained with our model illustrate clearly the point and it seems to us that the errors have been underestimated.

Finally let us point out that the various ratios studied in this paper have different types of dependence with respect to the form factor values at $q^2 = 0$ in the D sector.

- (i). \mathcal{A}_{LR} and \mathcal{A}'_{LR} depend only on $y^D(0)$.
- (ii). ρ_L and ρ'_L depend on $x^D(0)$ and $y^D(0)$.
- (iii). $R_{J/\Psi}$ and $R_{\Psi'}$ depend on $x^D(0)$, $y^D(0)$ and $z^D(0)$.
- (iv). R_{η_c} depends on $x^D(0)$ and $z^D(0)$.
- (v). S and T are independent of these three ratios.
- (vi). S^* and T^* depend on $x^D(0)$ and $y^D(0)$.

For the semi-leptonic normalized distribution $X(t)$, it is independent on these ratios in the $D \rightarrow \overline{K} + \ell^+ + \nu_\ell$ case and it depends on $x^D(0)$ and $y^D(0)$ in the $D \rightarrow \overline{K}^* + \ell^+ + \nu_\ell$ mode.

3°) In addition to the necessity of an improvement in accuracy for the observed rates – in particular, those involving the Ψ' are still badly known with errors at the 45% or 64% level – we need a detection of the $K + \eta_c$ and $K^* + \eta_c$ modes. On the polarization side, only the ratio ρ_L has been measured and it was an important quantity to be taken into account for the fits. It is clear that the knowledge of \mathcal{A}_{LR} , ρ'_L and \mathcal{A}'_{LR} predicted by our model would considerably help in reducing the size of the allowed domains in the parameter space.

The best situation would be to select only one scenario and a small domain in the parameter space. The worse situation for our model would be that the new measurements exclude the three presently remaining scenarios. Our approach being purely phenomenological is not connected with any theoretical calculation like, for instance, QCD sum rule or lattice gauge theories and our model is certainly not the unique way to compute $B \rightarrow K(K^*)$ hadronic form factors.

However if we are in the best situation previously mentioned, it will be necessary to provide a theoretical support to the so determined hadronic form factors and for that, results of Ref.[6] seem to be in a good shape because of the unusual q^2 behaviour prediction for $A_1(q^2)$.

If we are in the worse situation, it will be reasonable to think seriously of the role played by non-factorizable contributions.

Acknowledgements

Y. Y. K would like to thank the Commissariat à l'Energie Atomique of France for the award of a fellowship and especially G. Cohen-Tannoudji and J. Haïssinski for their encouragements.

Appendix A

1. Error on ρ_L (Eq:(III.17)).

$$\Delta\rho_L = 2\rho_L(1 - \rho_L) \left\{ \left[\frac{bx}{a - bx} \right]^2 \left(\frac{\Delta x^D}{x^D} \right)^2 + \left[\frac{2c^2 y^2}{1 + c^2 y^2} \right]^2 \left(\frac{\Delta y^D}{y^D} \right)^2 \right\}^{1/2} \quad (\text{A.1})$$

2. Error on $N = (a - bx)^2 + 2(1 + c^2 y^2)$

$$\Delta N = 2 \left\{ [bx(a - bx)]^2 \left(\frac{\Delta x^D}{x^D} \right)^2 + [2c^2 y^2]^2 \left(\frac{\Delta y^D}{y^D} \right)^2 \right\}^{1/2} \quad (\text{A.2})$$

3. Error on $R_{J/\Psi}$ (Eq:(III.19)).

$$\Delta R_{J/\Psi} = R_{J/\Psi} \left\{ \left(\frac{\Delta N}{N} \right)^2 + 4 \left(\frac{\Delta z^D}{z^D} \right)^2 \right\}^{1/2} \quad (\text{A.3})$$

4. From the PDG values in the D sector at $q^2 = 0$ [5].

$$\frac{\Delta x^D}{x^D} = \frac{15}{73} \quad , \quad \frac{\Delta y^D}{y^D} = \frac{25}{189} \quad , \quad \left(\frac{\Delta z^D}{z^D} \right)^2 = \left(\frac{1}{14} \right)^2 + \left(\frac{1}{25} \right)^2 \quad (\text{A.4})$$

5. In the Ψ' case, we make the situations $a \rightarrow a', b \rightarrow b', c \rightarrow c', x \rightarrow x', y \rightarrow y'$ and $z \rightarrow z'$.

6. Error on S (Eq:(VII.3)).

$$\Delta S = 2 S \left\{ \left(\frac{\Delta f_{J/\Psi}}{f_{J/\Psi}} \right)^2 + \left(\frac{\Delta f'_{\Psi}}{f'_{\Psi}} \right)^2 \right\}^{1/2} \quad (\text{A.5})$$

7. Error on S^* (Eq:(VII.4)).

$$\begin{aligned} \Delta S^* = 2 S^* \left\{ \left[\frac{\Delta S}{2S} \right]^2 + \left[\frac{bx(a-bx)}{N} - \frac{b'x'(a'-b'x')}{N'} \right]^2 \left(\frac{\Delta x^D}{x^D} \right)^2 \right. \\ \left. + 4 \left[\frac{c^2 y^2}{N} - \frac{c'^2 y'^2}{N'} \right]^2 \left(\frac{\Delta y^D}{y^D} \right)^2 \right\}^{1/2} \end{aligned} \quad (\text{A.6})$$

Appendix B

Semi-leptonic decay $D \rightarrow \bar{K} e^+ \nu_e$:

Using for the hadronic form factor $F_1^{DK}(q^2)$ a monopole q^2 -dependence, the decay rate is given by Eq.(IX.7) where the integral $I(\alpha_F)$ is defined in Eq.(IX.6). From the experimental rate, it is possible to deduce the value of $F_1^{DK}(0)$ if the slope parameter α_F is known.

The computation of $I(\alpha_F)$ has been done both numerically and analytically. In the case of interest here :

$$\frac{1}{(1+r)^2} < \alpha_F < \frac{1}{(1-r)^2} \quad , \quad r = \frac{m_K}{m_D} \quad (\text{B.1})$$

the result is :

$$\begin{aligned} I(\alpha_F) = & \frac{1-r^2}{2\alpha_F^3} \left\{ -6 + 9(1+r^2)\alpha_F - 2(1-r^2)^2\alpha_F^2 \right\} \\ & - \frac{3}{\alpha_F^4} \left\{ 1 - 2(1+r^2)\alpha_F + (1+r^4)\alpha_F^2 \right\} \ln r \\ & + \frac{3}{\alpha_F^4} \left[1 - (1+r^2)\alpha_F \right] \sqrt{[(1+r)^2\alpha_F - 1][1 - (1-r)^2\alpha_F]} \\ & \cdot \left\{ \frac{\pi}{2} - \text{Arctg} \frac{(1+r^2) - (1-r^2)^2\alpha_F}{(1-r^2)\sqrt{[(1+r)^2\alpha_F - 1][1 - (1-r)^2\alpha_F]}} \right\} \end{aligned} \quad (\text{B.2})$$

Of course such a formula can be used for any semi-leptonic decay of the type $\mathbf{0}^- \rightarrow \mathbf{0}^- + e^+ + \nu_e$ provided the corresponding form factor has a monopole q^2 -dependence.

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Figure Captions and Table Captions

A. Figure captions

1. **Figure 1** : Parameters for the $B \rightarrow K$ transitions. On the horizontal axis is Λ_F^2 , on the left vertical axis is the parameter μ , on the right vertical axis is the pole mass Λ_{DF}^2 . The relevant relations are

$$\mu^B = \frac{m_B^2 - m_K^2}{\Lambda_F^2} \quad , \quad \mu^D = \frac{\mu^B - \sigma}{1 + \sigma \mu^B} \quad , \quad \bar{\mu}^D = \frac{m_D^2 - m_K^2}{\Lambda_{DF}^2}$$

$$\Lambda_{DF}^2 = \frac{m_c}{m_b} \Lambda_F^2 - \left(1 - \frac{m_c}{m_b}\right) (m_b m_c - m_K^2)$$

The values of Λ_F and Λ_{DF} used by Bauer-Stech-Wirbel are indicated by the point BSW and they are $\Lambda_F = 5.43 \text{ GeV}$ and $\Lambda_{DF} = 2.11 \text{ GeV}$.

2. **Figure 2** : Parameters for the $B \rightarrow K^*$ transitions. On the horizontal axis is Λ_2^2 , on the left vertical axis is the parameter λ , on the right vertical axis is the pole mass Λ_{D2}^2 . The relevant relations are

$$\lambda^B = \frac{m_B^2 - m_{K^*}^2}{\Lambda_2^2} \quad , \quad \lambda^D = \frac{\lambda^B - \sigma}{1 + \sigma \lambda^B} \quad , \quad \bar{\lambda}^D = \frac{m_D^2 - m_{K^*}^2}{\Lambda_{D2}^2}$$

$$\Lambda_{D2}^2 = \frac{m_c}{m_b} \Lambda_2^2 - \left(1 - \frac{m_c}{m_b}\right) (m_b m_c - m_{K^*}^2)$$

The values of Λ_2 and Λ_{D2} used by Bauer-Stech-Wirbel are indicated by the point BSW and they are $\Lambda_2 = 5.82 \text{ GeV}$ and $\Lambda_{D2} = 2.53 \text{ GeV}$.

3. **Figure 3** : The quark level colour-suppressed diagram $\bar{b}q \rightarrow (\bar{c}c) + (\bar{s}q)$.
4. **Figure 4** : $A_2^{BK^*}(m_{\Psi'}^2)$ and $x^B(m_{\Psi'}^2)$ as functions of Λ_2 for $5 \text{ GeV} \leq \Lambda_2 \leq 6 \text{ GeV}$. One standard deviations are indicated with dotted points.
5. **Figure 5** : The fractional longitudinal polarization ρ'_L as a function of Λ_2 for $5 \text{ GeV} \leq \Lambda_2 \leq 6 \text{ GeV}$. One standard deviations are indicated with dotted points. Our scenario-independent upper limit $\rho'_L \leq 0.5664$ is indicated.

6. **Figure 6** : The hadronic form factor $F_1^{BK}(q^2)$ at $q^2 = 0$, $q^2 = m_{J/\Psi}^2$ and $q^2 = m_{\Psi'}^2$ as functions of Λ_F for $5 \text{ GeV} \leq \Lambda_F \leq 6 \text{ GeV}$. One standard deviations are indicated with dotted points.
7. **Figure 7** : The ratio of rates $R_{\Psi'}$ as a function of Λ_2 and Λ_F for $5 \text{ GeV} \leq \Lambda_2, \Lambda_F \leq 6 \text{ GeV}$. The cross points indicate the one standard deviation experimental limits $0.44 \leq R_{\Psi'} \leq 3.62$.
8. **Figure 8** : The allowed domain in the $\Lambda_1, \Lambda_2, \Lambda_V$ space due to the constraints $\rho_L + \Delta\rho_L \geq 0.70$ for $\Lambda_2, \Lambda_V \in (5 - 6) \text{ GeV}$, $\Lambda_1 \geq 5 \text{ GeV}$. The scenario is $[n_1, n_2, n_V] = [-1, 2, 2]$.
9. **Figure 9** : Same as Figure 8 for the scenario $[-1, 1, 2]$.
10. **Figure 10** : Same as Figure 8 for the scenario $[-1, 0, 2]$.
11. **Figure 11** : Same as Figure 8 for the scenario $[-1, 2, 1]$.
12. **Figure 12** : The allowed domain in the $\Lambda_1, \Lambda_2, \Lambda_V$ space due to the constraints $\rho_L + \Delta\rho_L \geq 0.70$ and $R_{J/\Psi} - \Delta R_{J/\Psi} \leq 2.20$ for $\Lambda_2, \Lambda_V \in (5 - 6) \text{ GeV}$, $\Lambda_1 \geq 5 \text{ GeV}$. The scenario is $[n_1, n_2, n_V] = [-1, 2, 2]$. $\Delta\rho_L$ and $\Delta R_{J/\Psi}$ are theoretical errors induced by experimental errors of $x^D(0), y^D(0)$ and $z^D(0)$.
13. **Figure 13** : Same as Figure 12 for the scenario $[-1, 1, 2]$.
14. **Figure 14** : Same as Figure 12 for the scenario $[-1, 0, 2]$.
15. **Figure 15** : The allowed domain in the $\Lambda_F, \Lambda_2, \Lambda_V$ space due to the constraint $\Lambda_{1, MIN}(\Lambda_2, \Lambda_V, \Lambda_F = 5 \text{ GeV}) \leq \Lambda_1 \leq \Lambda_{1, MAX}(\Lambda_2, \Lambda_V)$ with $\Lambda_F \geq 5 \text{ GeV}$. The scenario is $[n_1, n_2, n_V] = [-1, 2, 2]$.
16. **Figure 16** : Same as Figure 15 for the scenario $[-1, 1, 2]$.
17. **Figure 17** : Same as Figure 15 for the scenario $[-1, 0, 2]$.
18. **Figure 18** : The normalized dimensionless distribution $X(t)$ for the semi-leptonic decay $D \rightarrow \bar{K} + \ell^+ + \nu_\ell$. The scenario $n_2 = 2$ corresponds to $5 \text{ GeV} \leq \Lambda_F \leq 5.71 \text{ GeV}$, the scenarios $n_2 = 1$ corresponds to $5 \text{ GeV} \leq \Lambda_F \leq 5.39 \text{ GeV}$ and the scenario $n_2 = 0$

corresponds to $5 \text{ GeV} \leq \Lambda_F \leq 5.10 \text{ GeV}$. By Eq.(II.24) the pole masses Λ_{DF} in the D sector can be obtained from Λ_F given here.

19. **Figure 19** : The normalized dimensionless distribution $X(t)$ for the semi-leptonic decay $D \rightarrow \overline{K}^* + \ell^+ + \nu_\ell$. The scenario is $[n_1, n_2, n_V] = [-1, 2, 2]$. The thickness of the curve is due to the $\Lambda_1, \Lambda_2, \Lambda_V$ ranges.
20. **Figure 20** : Same as Figure 19 for the scenario $[-1, 1, 2]$.
21. **Figure 21** : Same as Figure 19 for the scenario $[-1, 0, 2]$.

B. Table Captions

1. **Table 1 :**

Experimental data of the decay rates for $K^{(*)} + \Psi'$ and $K^{(*)} + J/\Psi$ as averaged by PDG [5].

2. **Table 2 :**

Experimental data and the averaged values for the ratios $R_{\Psi'}$, $R_{J/\Psi}$.

3. **Table 3 :**

Numerical results for ρ_L , $\rho_L + \Delta\rho_L$, $R_{J/\Psi}$ and $R_{J/\Psi} - \Delta R_{J/\Psi}$ at three values of Λ_1 ; $\Lambda_{1, MAX}$, $\Lambda_{1, MIN}$ and an intermediate value between two extreme values in three scenarios ($n_2 = 2, 1, 0$).

4. **Table 4 :**

Experimental data and the averaged values for the ratios S , S^* .

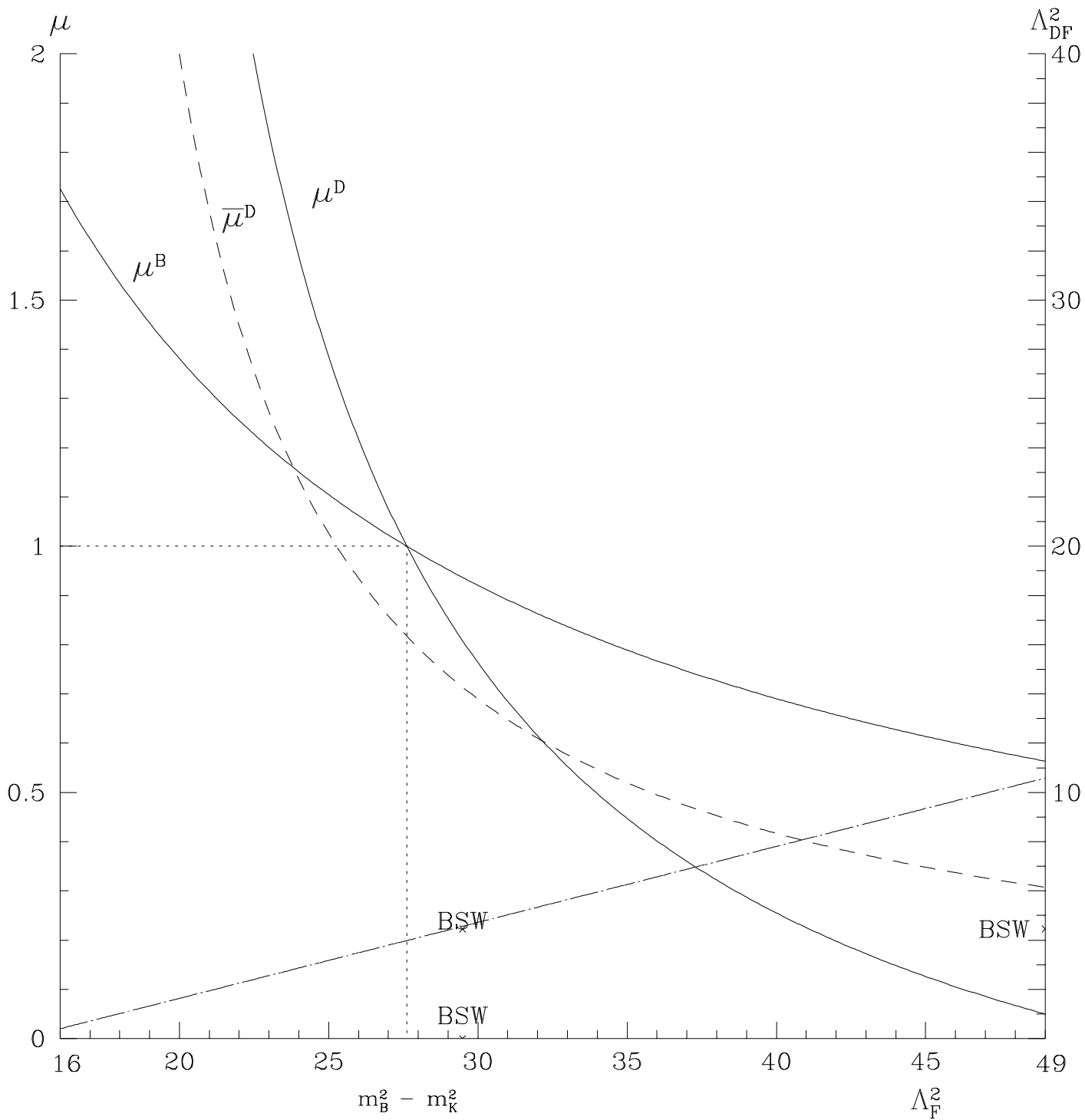


Figure 1

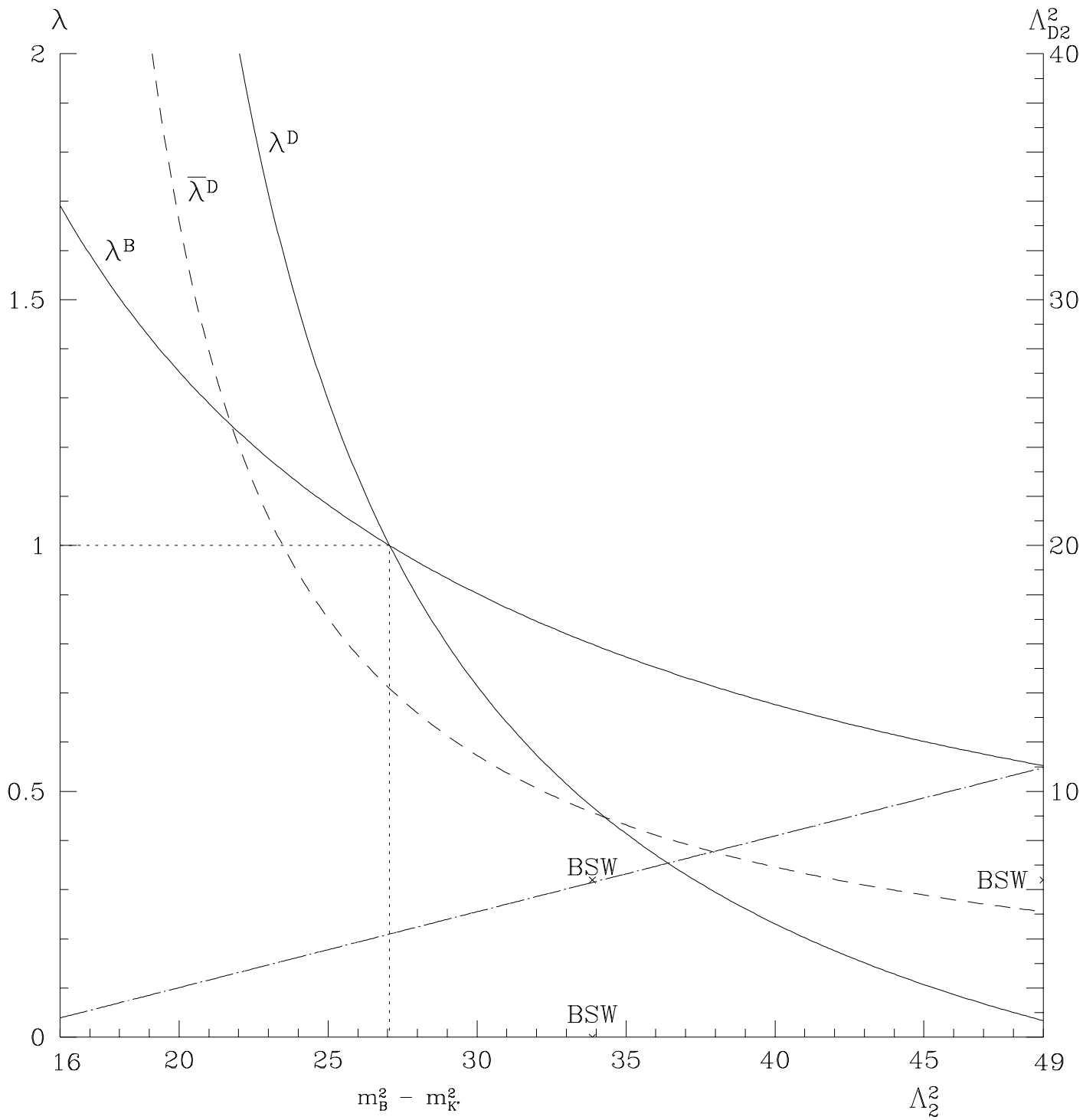


Figure 2

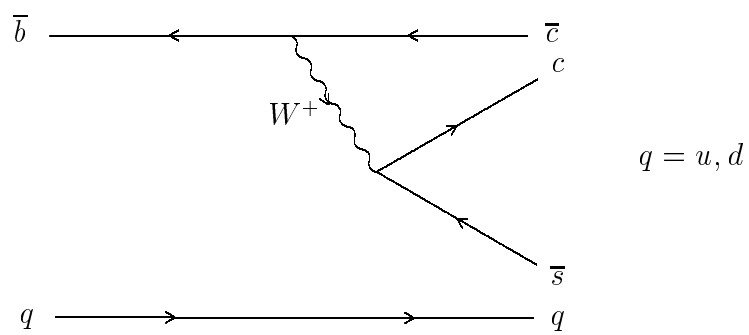


Figure 3

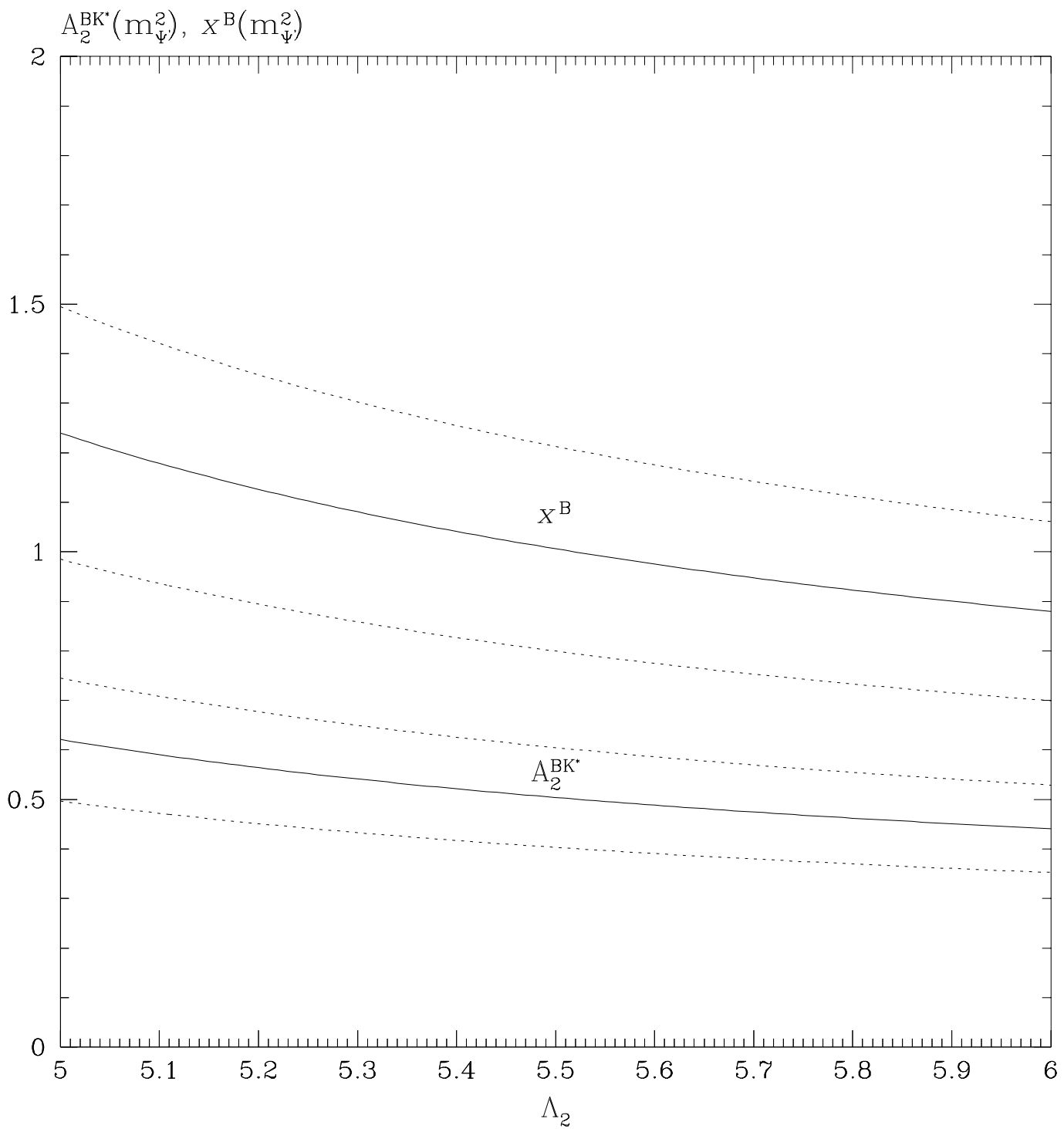


Figure 4

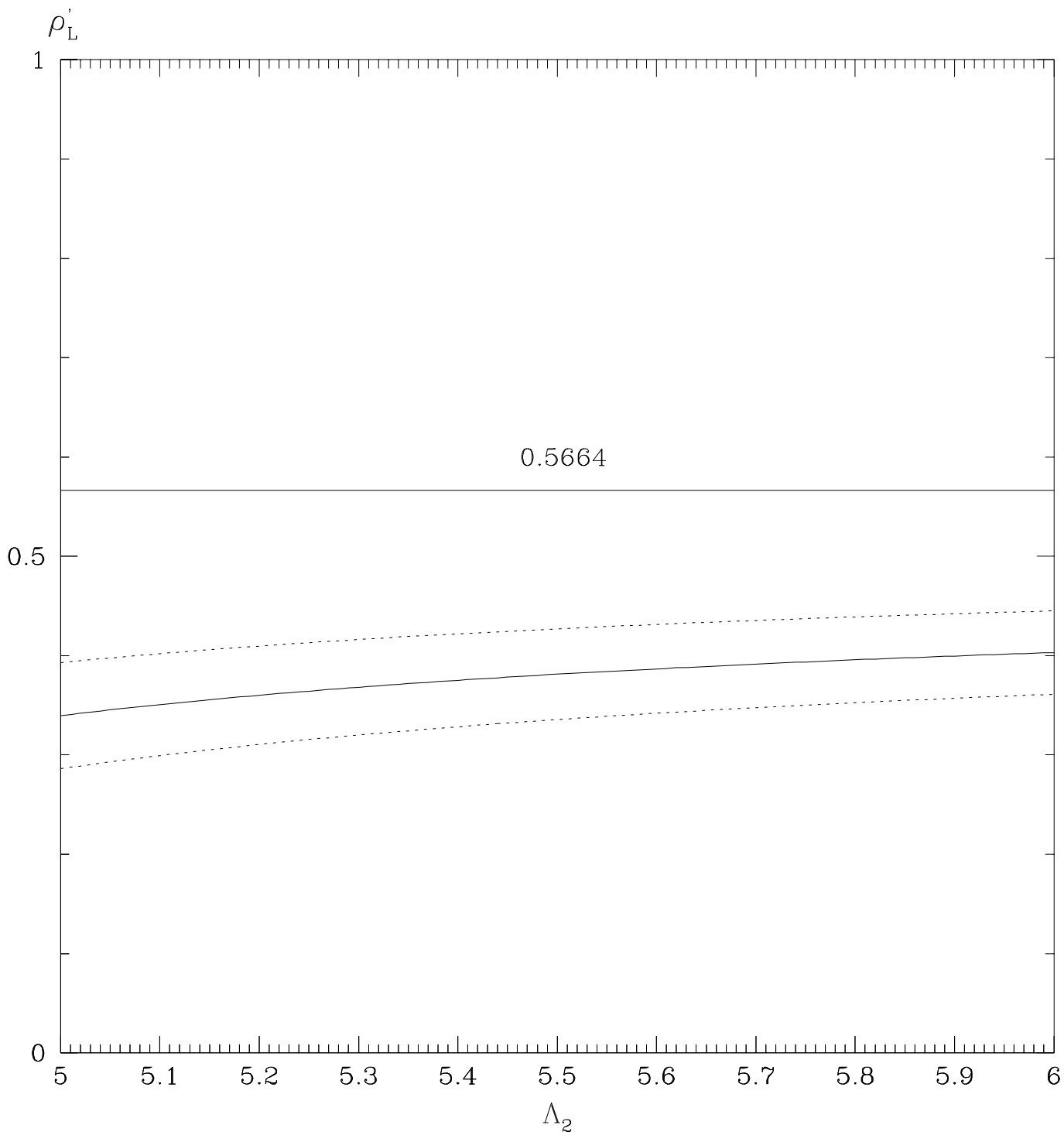


Figure 5

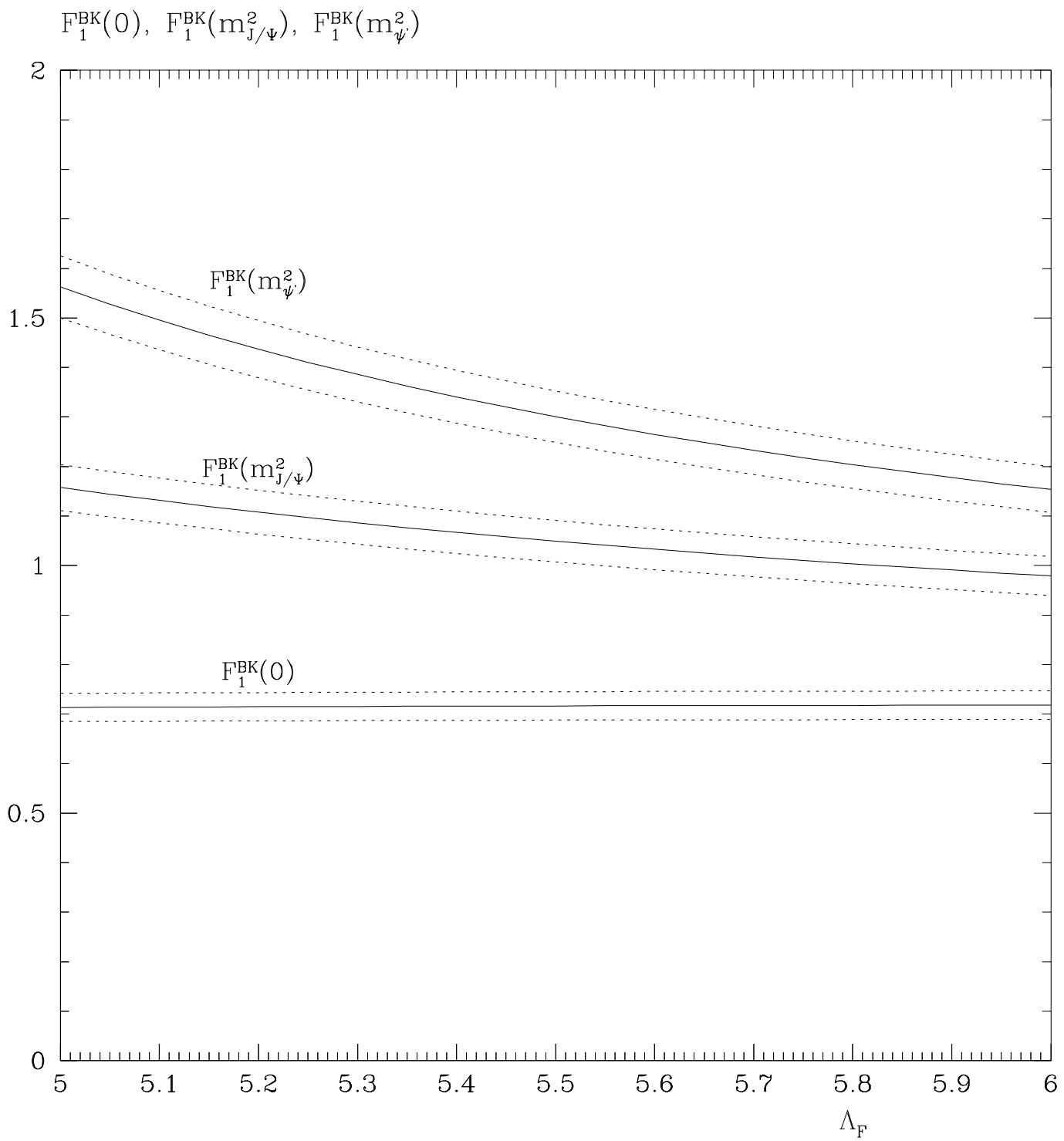


Figure 6

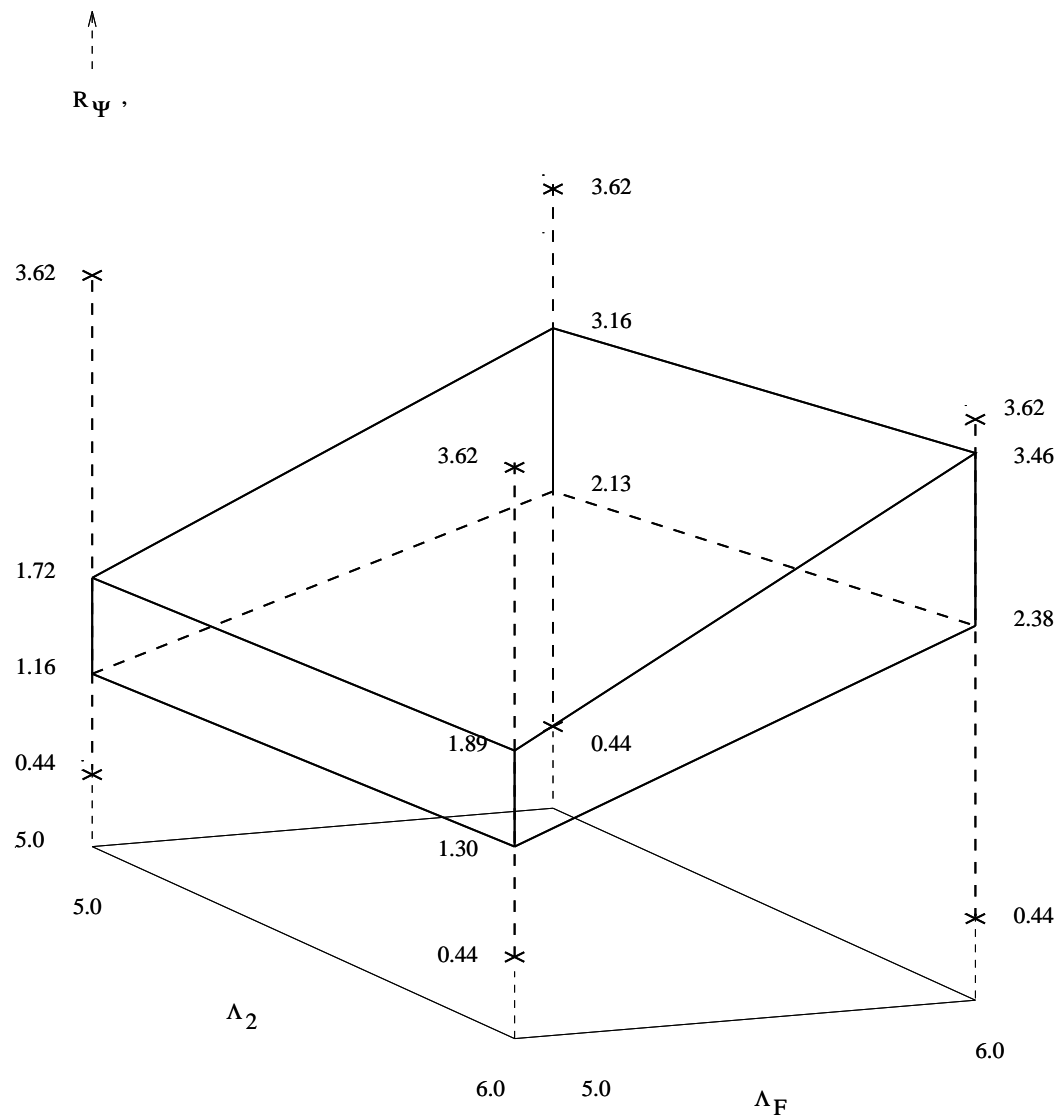


Figure 7

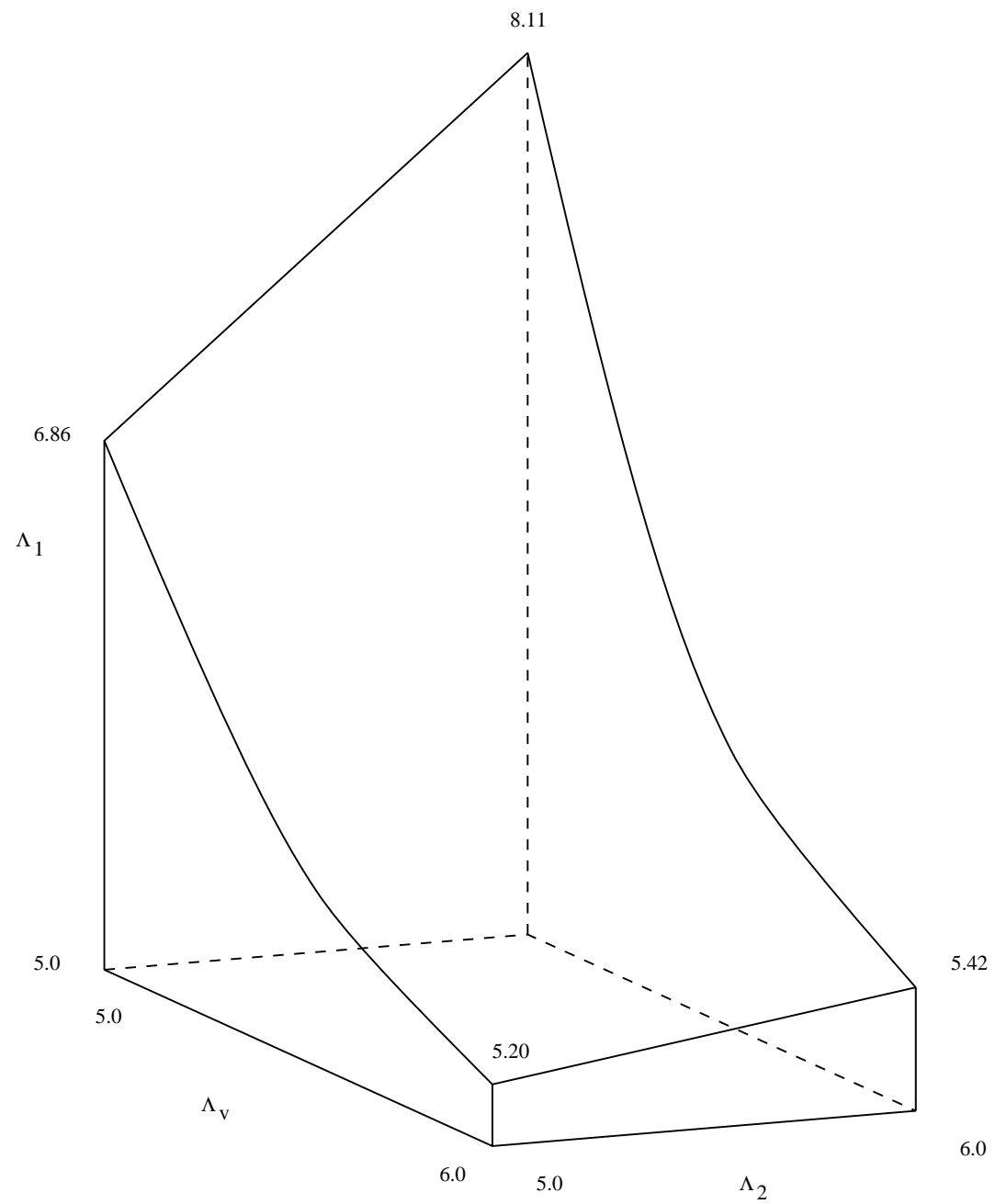


Figure 8

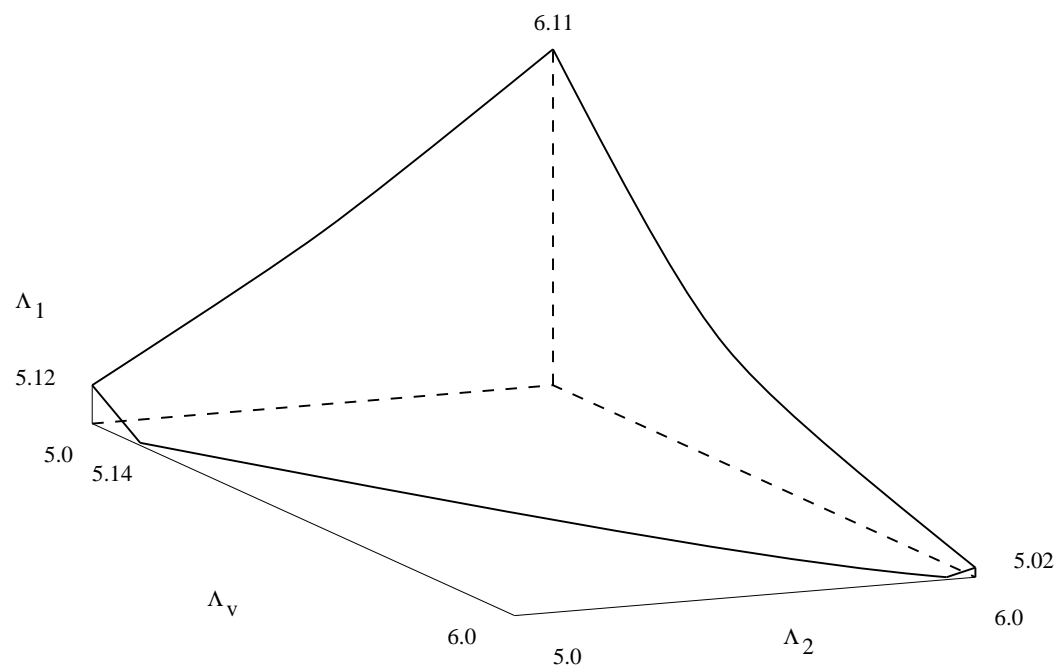


Figure 9

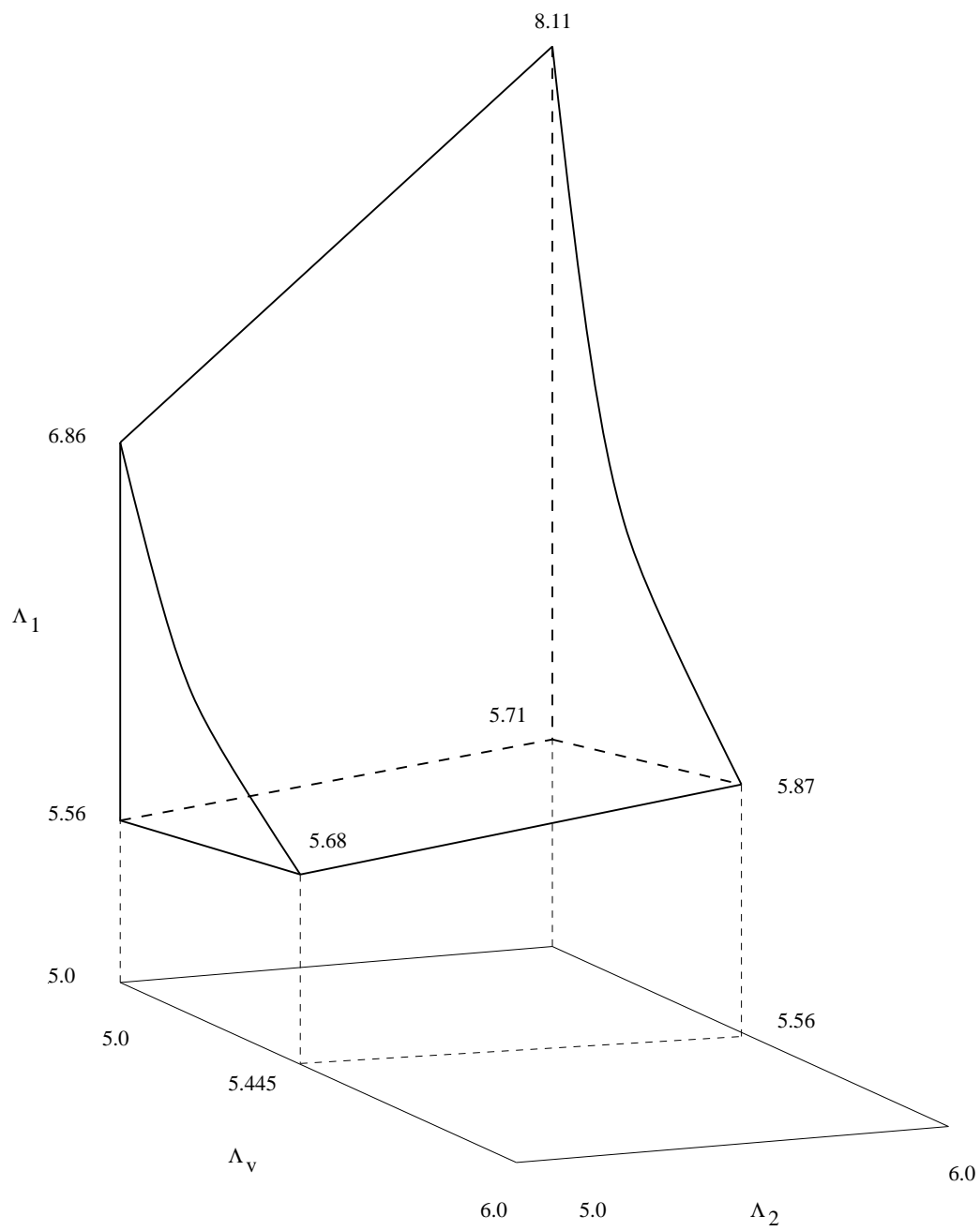


Figure 12

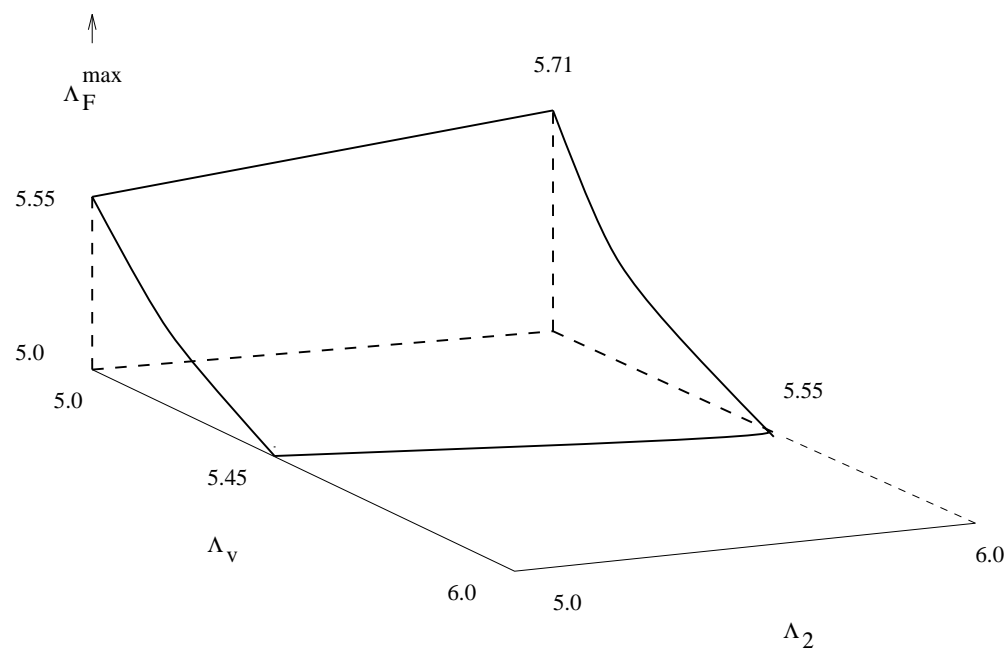


Figure 15

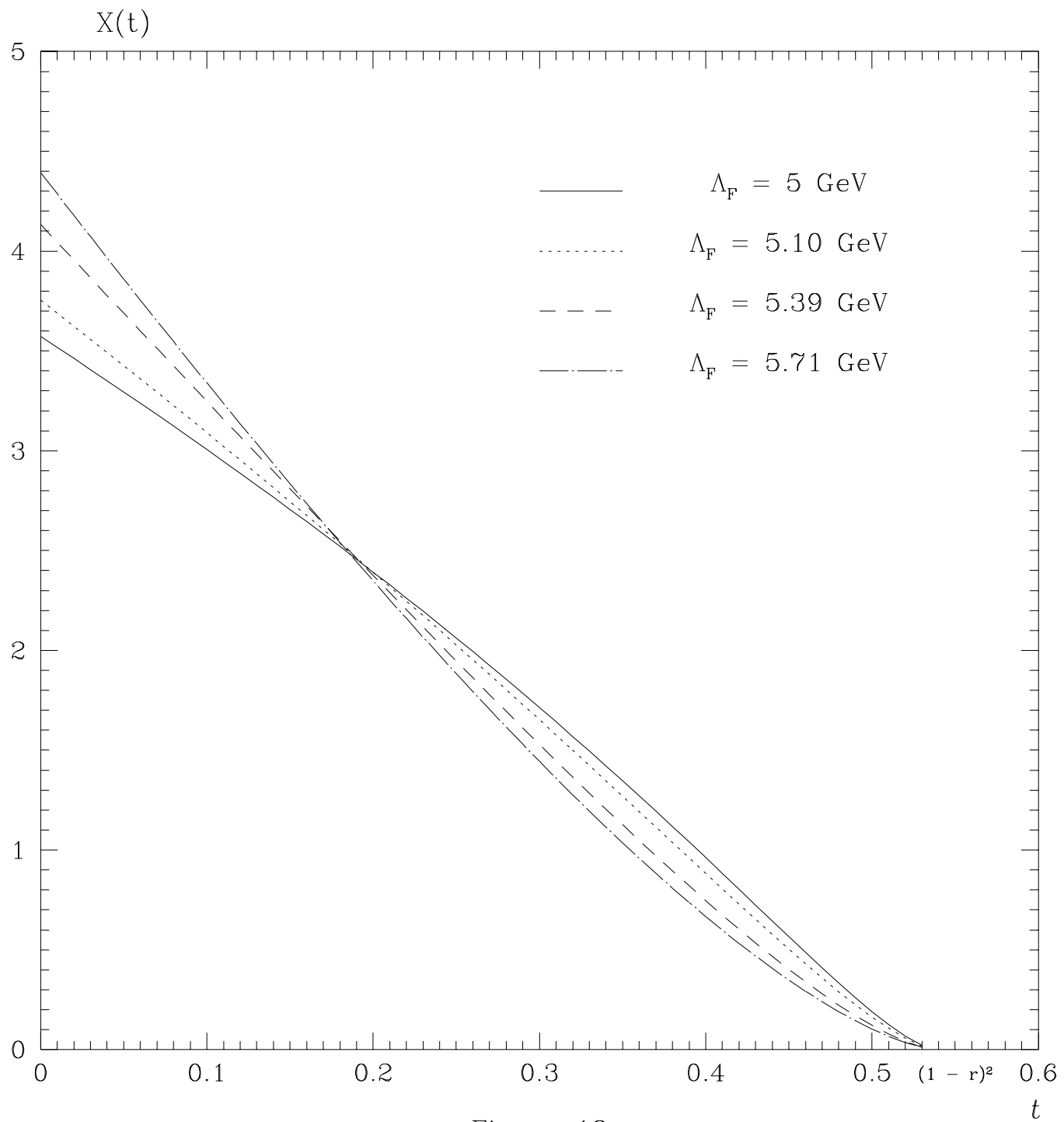


Figure 18

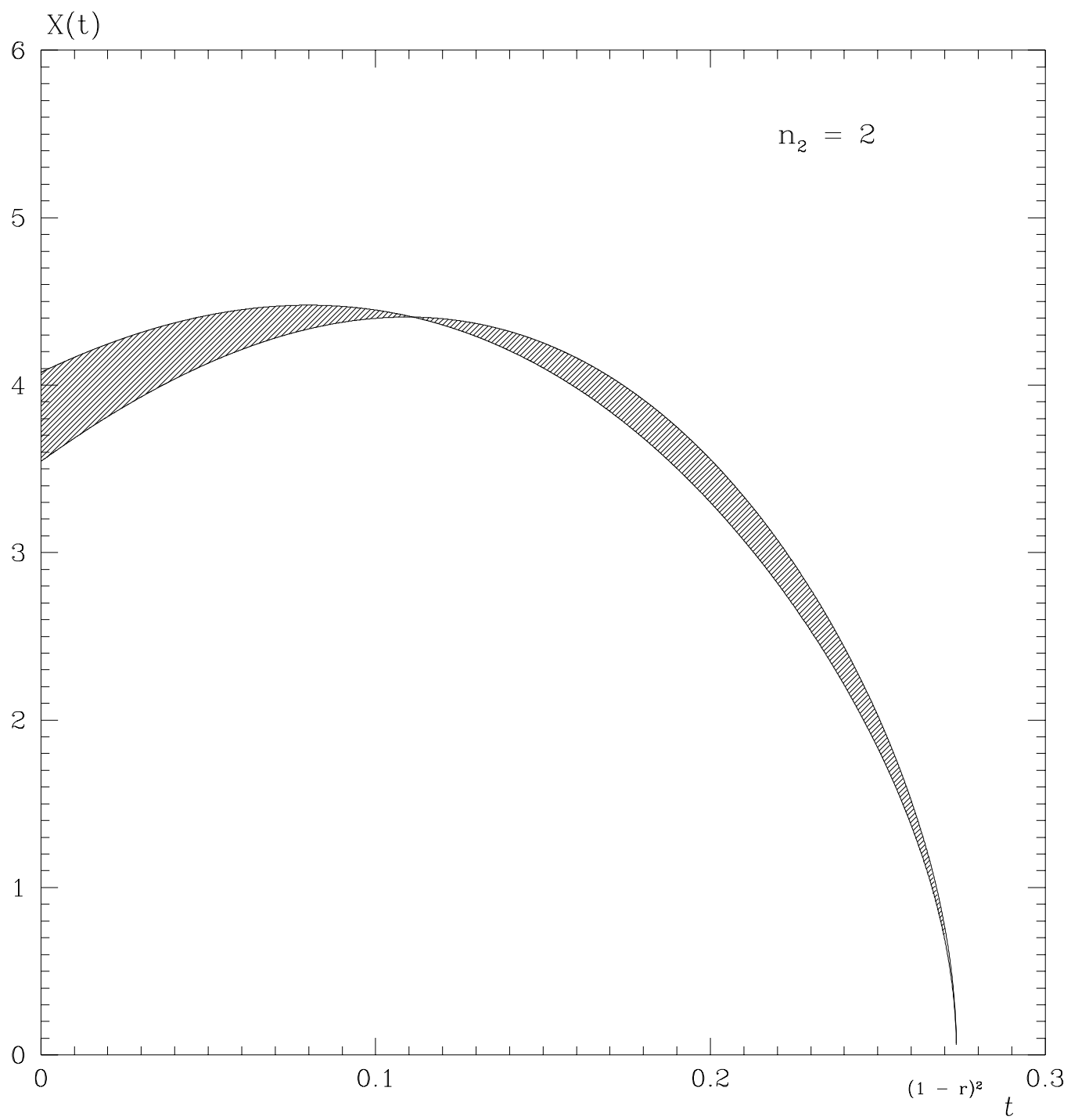


Figure 19

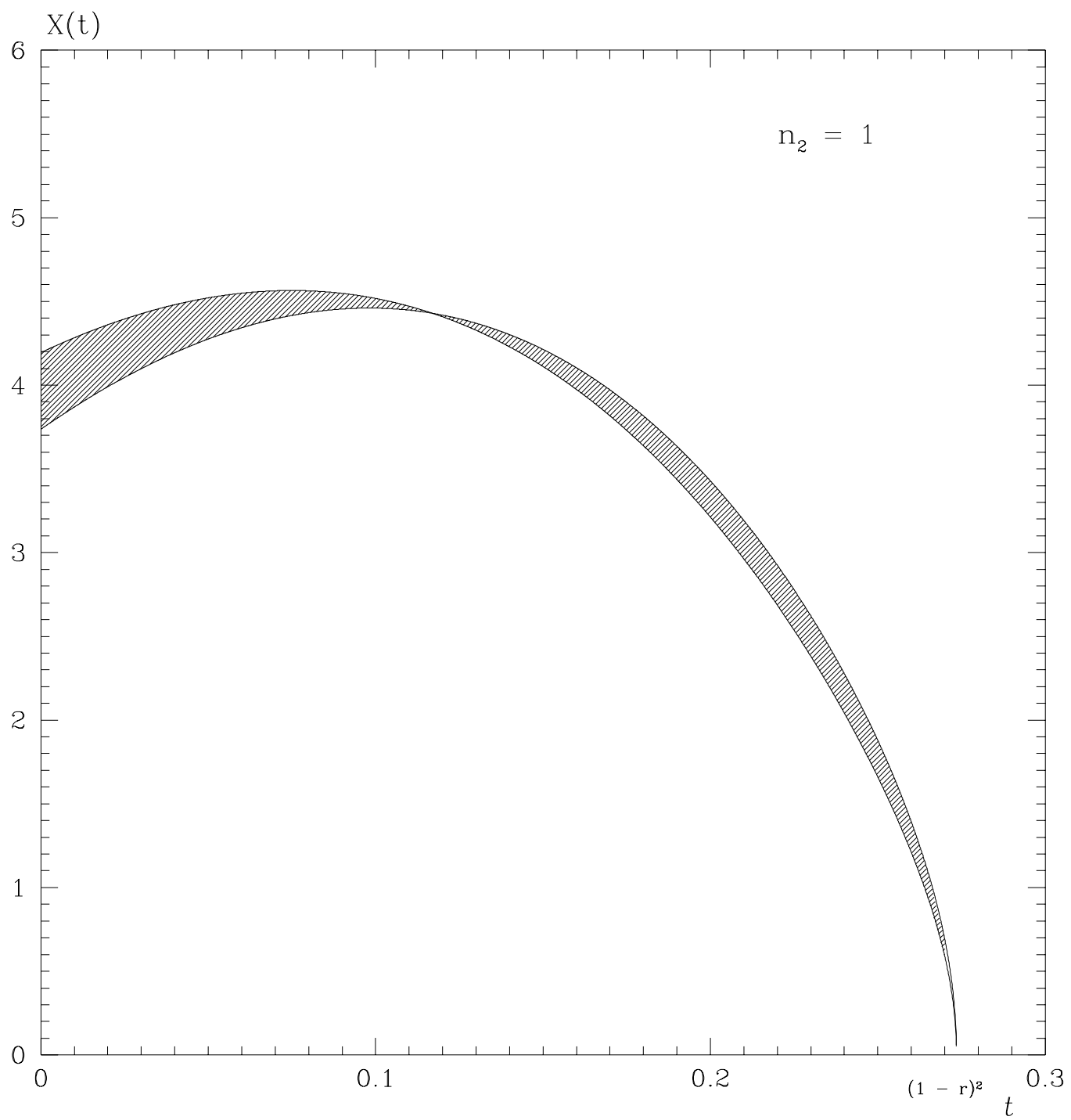


Figure 20

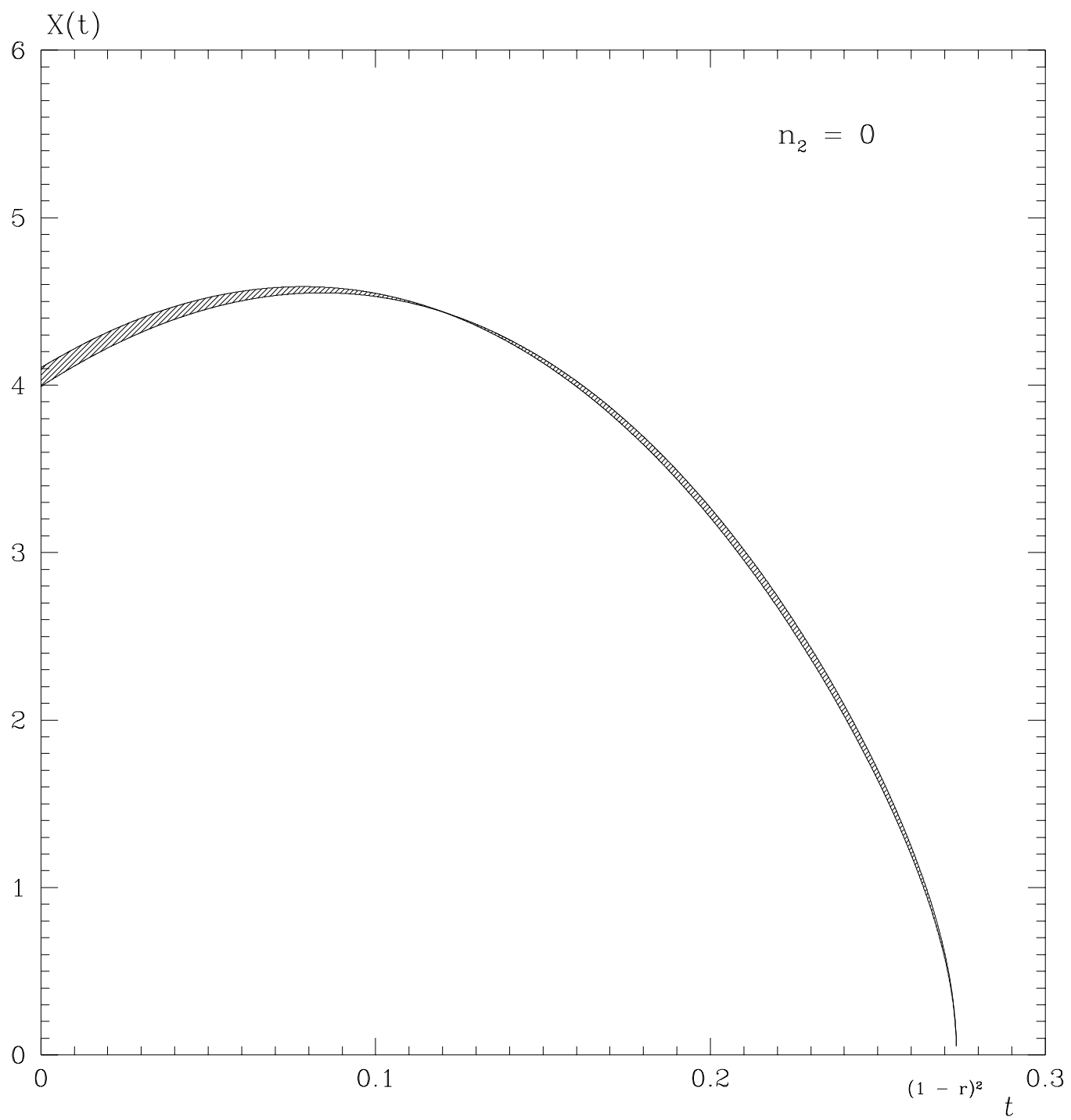


Figure 21

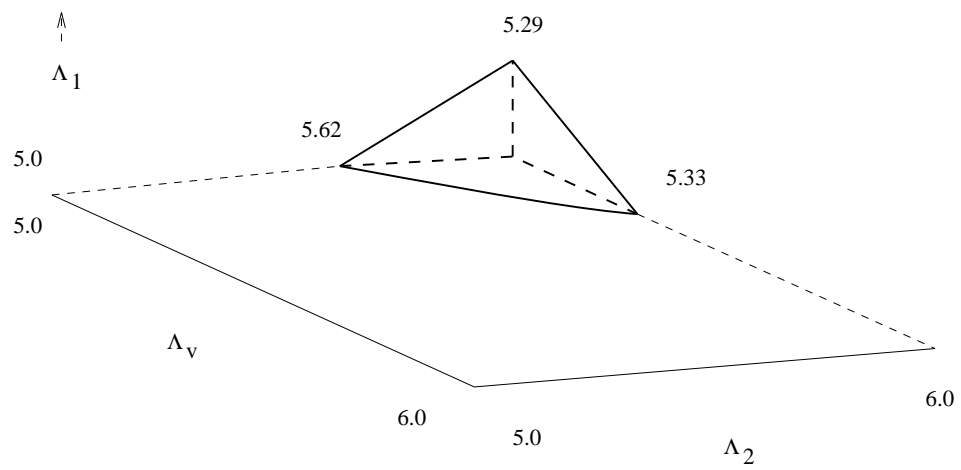


Figure 10

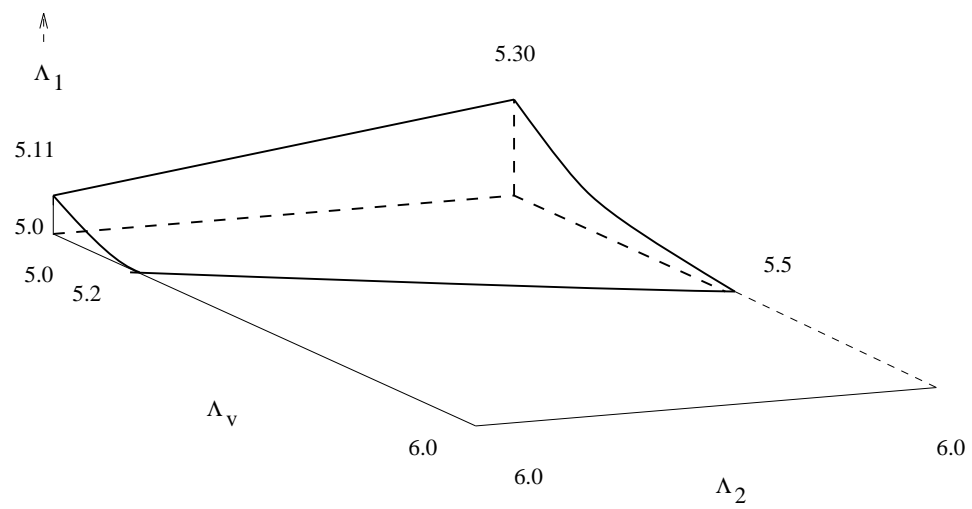


Figure 11

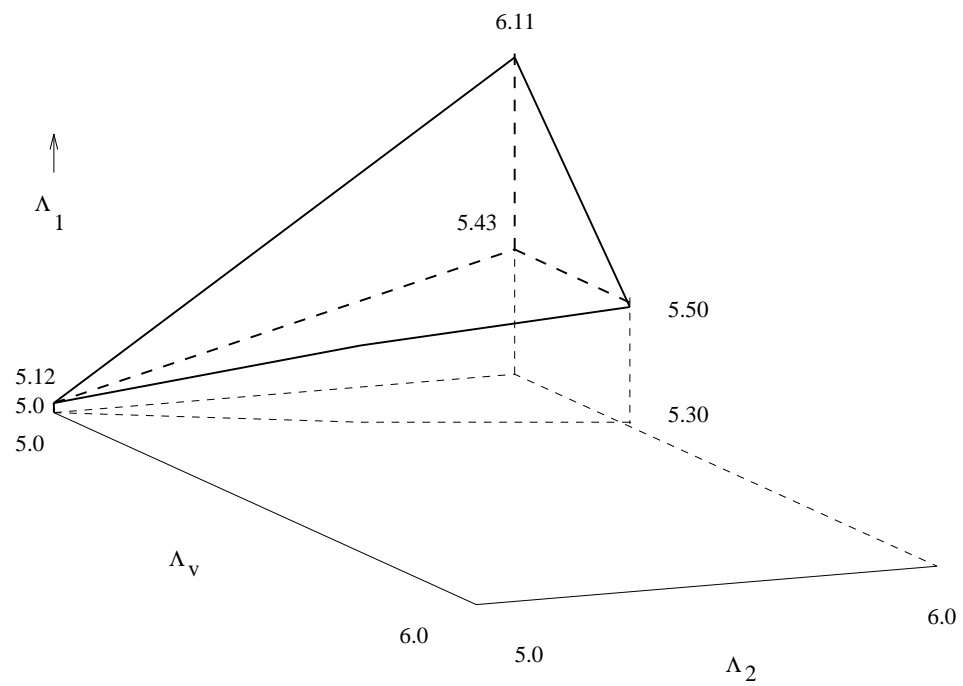


Figure 13

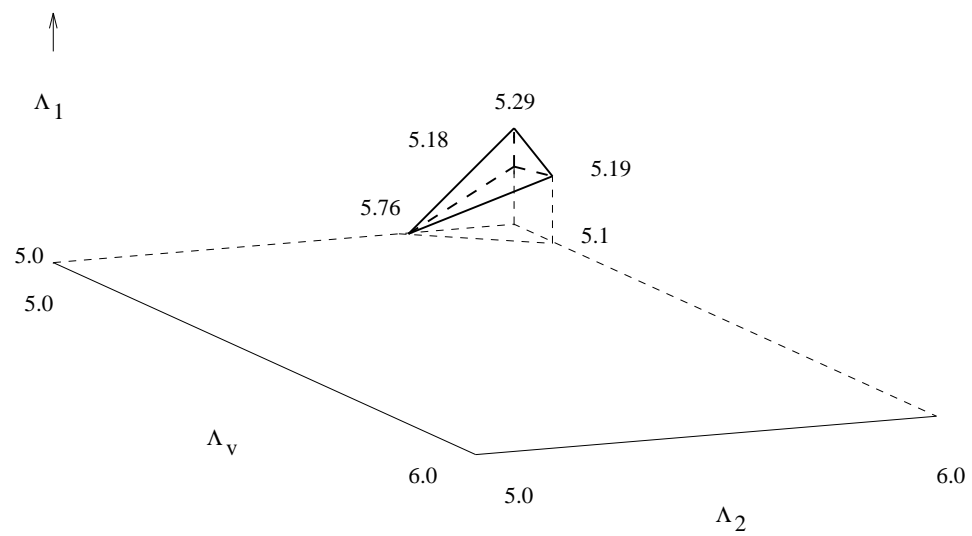


Figure 14

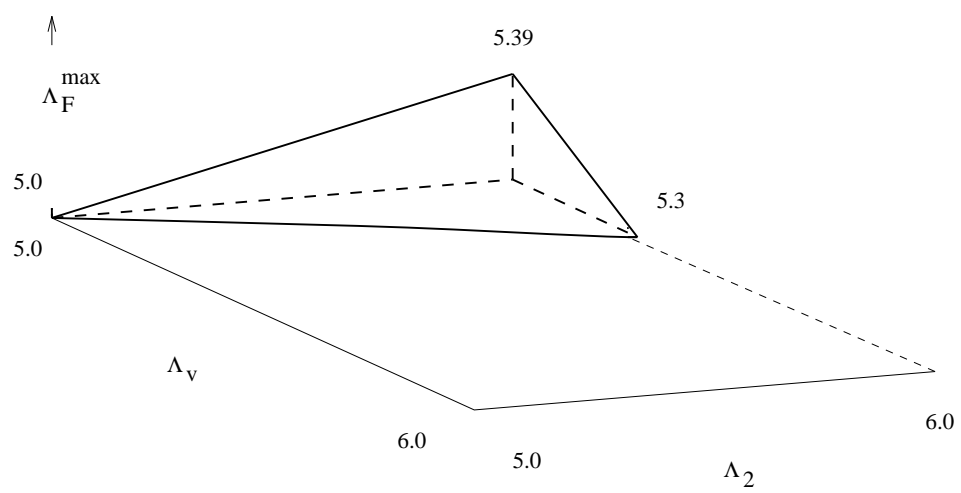


Figure 16

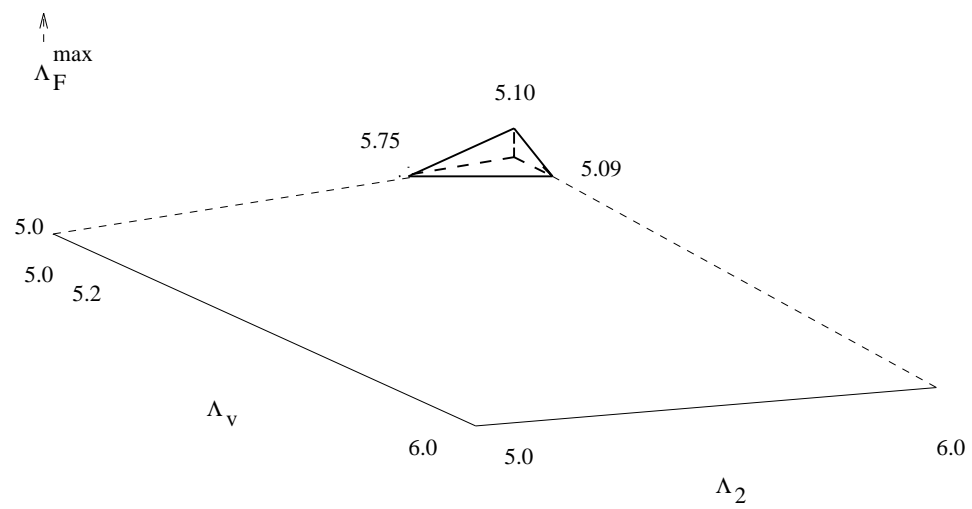


Figure 17